

New prime K-tuple theorem (3)

$$P, jP + j + 1 (j = 1, \dots, k)$$

Jiang, Chunxuan (蒋春暄)

Institute for Basic Research, Palm Harbor, FL34682-1577, USA

And: P. O. Box 3924, Beijing 100854, China (蒋春暄, 北京 3924 信箱, 100854)

jiangchunxuan@sohu.com, cxjiang@mail.bcf.net.cn, jcxuan@sina.com, Jiangchunxuan@vip.sohu.com,
jcxxxx@163.com

Abstract: Using Jiang function we prove that for every positive integer k there exist infinitely many primes P such that each of $jP + j + 1$ is prime.

[Chun-Xuan, Jiang. **New prime K-tuple theorem (3)** $P, jP + j + 1 (j = 1, \dots, k)$. *Academ Arena* 2016;8(2s): 5-5]. (ISSN 1553-992X). <http://www.sciencepub.net/academia>. 3. doi:[10.7537/marsaj0802s1603](https://doi.org/10.7537/marsaj0802s1603).

Keywords: new; prime; k -tuple; theorem; Jiang Chunxuan; mathematics; science; number; function

Theorem

$$P, jP + j + 1 (j = 1, \dots, k) \quad (1)$$

For every positive integer k there exist infinitely many primes P such that each of $jP + j + 1$ is prime.

Proof. We have Jiang function [1, 2]

$$J_2(\omega) = \prod_P (P - 1 - \chi(P)) \quad (2)$$

$$\omega = \prod_P P$$

where

$\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^k (jq + j + 1) \equiv 0 \pmod{P} \quad (3)$$

where $q = 1, \dots, P - 1$

From (3) we have

If $P \leq k + 1$ then $\chi(P) = P - 2$, if $k + 1 < P$ then $\chi(P) = k$

From (3) and (2) we have

$$J_2(\omega) = \prod_{k+1 < P} (P - k - 1) \neq 0 \quad (4)$$

We prove that for every positive integer k there exist infinitely many primes P such that each of $jP + j + 1$ is prime.

We have the best asymptotic formula [1, 2]

$$\pi_{k+1}(N, 2) = \left| \left\{ P \leq N : jP + j + 1 = \text{prime} \right\} \right| \sim \frac{J_2(\omega) \omega^k}{\phi^{k+1}(\omega) \log^{k+1} N} \quad (5)$$

The author takes a day to write this paper.

References

1. Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. (<http://www.wbabin.net/math/xuan2.pdf>) (<http://vixra.org/pdf/0812.0004v2.pdf>)
2. Chun-xuan Jiang, The Hardy-Littlewood prime k -tuple conjecture is false. <http://www.wbabin.net/math/xuan77.pdf>