Queueing Theory in Practice

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Abstract: This paper will take a brief look into the Stochastic modeling of queuing theory along with examples of the models and applications of their use. The goal of the paper is to provide the reader with enough background in order to properly model a basic queuing system into one of the categories we will look at, when possible.
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Queueing theory is considered to be a branch of operations research. It constitutes a powerful tool in modelling and performance analysis of many complex systems, such as computer networks, telecommunication systems, call centres, flexible manufacturing systems and service systems. Recently, the queueing theory including queueing systems and networks arouses mathematicians', engineers' and economics interests.

Queueing theory evolved originally out of an investigation of problems dealing with the design of telephone systems. Now, some 70 years later, we are witnessing a tremendous accumulation of theoretical results for "idealized" systems that apparently have not been as effective in dealing with other types of real-life problems. Queueing theory differs from other mathematical techniques of Operations Research in that it does not deal with optimization models. Rather, it utilizes mathematical analysis to determine the system's measures of effectiveness such as the expected waiting time per customer and the facility's percentage of idle time. These measures are then used as data in the context of an optimization (cost) model for determining the system's capacity. The obstacles in applying queueing theory to practical problems occur both in modeling the system mathematically and in determining its optimum design parameters. This paper identifies the areas of application in terms of their amenability to analysis by queueing theory, and provides suggestions that can enhance the applicability of queueing models in real life.

A stochastic process is a family of random variables ***X*** where ***t*** is a parameter running over a suitable index set ***T***. (Where convenient, we will write ***X(t)*** instead of ***X***,.) In a common situation, the index ***t*** corresponds to discrete units of time, and the index set is ***T*** = {***0, 1, 2, . . .***}. In this case, ***X***, might represent the outcomes at successive tosses of a coin, repeated responses of a subject in a learning experiment, or successive observations of some characteristics of a certain population. Stochastic processes for which ***T*** = ***[0, c)*** are particularly important in applications. Here ***t*** often represents time, but different situations also frequently arise. For example, ***t*** may represent distance from an arbitrary origin, and ***X***, may count the number of defects in the interval (***0, t***] along a thread, or the number of cars in the interval (***0, t***] along a highway. Stochastic processes are distinguished by their state space, or the range of possible values for the random variables ***X*** by their index set ***T***, and by the dependence relations among the random variables ***X***,.

Events and Probabilities

Let ***A*** and ***B*** be events. The event that at least one of ***A*** or ***B*** occurs is called the union of ***A*** and ***B*** and is written ***A U B***; the event that both occur is called the intersection of ***A*** and ***B*** and is written $A \cap B$, or simply ***AB.*** This notation extends to finite and countable sequences of events. Given events $A_1, A_2, \ldots\ldots,$ the event that at least one occurs is written $A_1 \cup A_2 \cup \ldots \cup_{i=1} A_i$ the event that all occur is written $A_1 \cap A_2 \cap \ldots \cap_{i=1} A_i$ The probability of an event ***A*** is written $P_r(A)$. The certain event, denoted by Ω, always occurs, and $P_r(\Omega) = 1$. The impossible event, denoted by ϕ, never occurs, and $P_r(\phi) = 0$. It is always the case that $0 \le P_r(A) \le 1$ for any event ***A***.

Events ***A, B*** are said to be disjoint if $A \cap B = \phi$; that is, if ***A*** and ***B*** cannot both occur. For disjoint events ***A, B*** we have the addition law $P_r(A \cup B) = P_r(A) + P_r(B)$. A stronger form of the addition law is as follows: Let $A_1, A_2, \ldots\ldots,$ be events with A_i; and A_j; disjoint whenever $i \neq j$.

Then $P_r\left(\bigcup_{i=1} A_i\right)=\sum_{i=1}^{\infty} P_r\left(A_i\right)$. The addition law leads directly to the law of total probability:

Random Variables

Most of the time we adhere to the convention of using capital letters such as X, Y, Z to denote random variables, and lowercase letters such as x, y, z for real numbers. The expression (X:5 x) is the event that the random variable X assumes a value that is less than or equal to the real number ***x***. This event may or may not occur, depending on the outcome of the experiment or phenomenon that determines the value for the random variable X. The probability that the event occurs is written Pr{***X*** <- ***x***). Allowing x to vary, this probability defines a function

$$F(x)=P_r(X \le x), \ -\infty < x < +\infty$$

called the distribution function of the random variable ***X***.

Exponential, Moments and Poisson Probability Distributions:

The *mean* (or the expectation) of a discrete random variable is defined by

$$E(X)=\sum nP(x)$$

Equivalently, the mean of a continuous random variable is defined by

$$E(X)=\int_{-\infty}^{\infty} x\ f(x)dx$$

The exponential distribution with parameter λ is given by $\lambda e^{-\lambda t}$ for $t>0$. If T is a random variable that represents interarrival times with the exponential distribution, then $P(T\le t)=1-e^{-\lambda t}\ \ and\ \ P(T>t)=e^{-\lambda t}$. This distribution lends itself well to modeling customer interarrival times or service times for a number of reasons. The first is the fact that the exponential function is a strictly decreasing function of t. This means that after an arrival has occurred, the amount of waiting time until the next arrival is more likely to be small than large. Another important property of the exponential distribution is what is known as the no-memory property. The no-memory property suggests that the time until the next arrival will never depend on how much time has already passed. This makes intuitive sense for a model where we're measuring customer arrivals because the customers' actions are clearly independent of one another.

The Input Process:

To begin modeling the input process, we define t_i as the time when the ith customer arrives. For all $i \ge 1$, we define $T_i = t_{i+1} - t_i$ to be the ith interarrival time. We also assume that all T_i**'s** are independent, continuous random variables, which we represent by the random variable ***A*** with probability density ***a(t)***. Typically, ***A*** is chosen to have an exponential probability distribution with parameter λ defined as the arrival rate, that is to say, $a(t)=\lambda e^{-\lambda t}$.

The Output Process:

Much like the input process, we start analysis of the output process by assuming that service times of different customers are independent random variables represented by the random variable ***S*** with probability density $s(t) = \mu e^{-\mu t}$. We also define μ as the service rate, with units of customers per hour. Ideally, the output process can also be modeled as an exponential random variable, as it makes calculation much simpler.

Birth-Death Processes:

We define the number of people located in a queuing system, either waiting in line or in service, to be the state of the system at time ***t***. At $t = 0$, the state of the system is going to be equal to the number of people initially in the system. The initial state of the system is noteworthy because it clearly affects the state at some future ***t***. Knowing this, we can define $P_{ij}(t)$ as the probability that the state at time ***t*** will be ***j***, given that the state at $t = 0$ was ***i***. For a large ***t***, $P_{ij}(t)$ will actually become independent of ***i*** and approach a limit π_j .This limit is known as the steady-state of state ***j***.

Conclusion: This paper gives a general look at the queueing theory. Presented queueing With the knowledge of probability theory, input and output models, and birthdeath processes, it is possible to derive many different queuing models, including but not limited to the ones.

Reference

1. S. Stidham, "Analysis, design and control of queueing systems", Operations Research 50 (1), 197–216 (2002).
2. Wayne L Winston, Operations Research: Applications and Algorithms, 2nd edition, PWS-Kent Publishing, Boston, 1991.

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