New prime k-tuple theorem（2）

$P, P + j(j+1)(j = 1, \cdots, k)$

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Abstract: Using Jiang function we prove that for every positive integer *k* there exist infinitely many primes P such that each of $P + j(j+1)$ is prime.

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Theorem.

$$P, P + j(j+1)(j = 1, \cdots, k). \qquad (1)$$

For every positive integer *k* there exist infinitely many primes P such that each of $P + j(j+1)$ is prime.

Proof. We have Jiang function [1, 2, 3]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)], \qquad (2)$$

where $\omega = \prod_P P$,

$\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k} [q + j(j+1)] \equiv 0 \pmod P, \qquad (3)$$

where $q = 1, \cdots, P-1$.

From (3) we have

If $P < 2k$ then $\chi(P) = \frac{P-1}{2}$, If $2k < P$ then $\chi(P) = k$.

From (3) and (2) we have.

$$J_2(\omega) = \prod_{P=3}^{P<2k} \frac{P-1}{2} \prod_{2k<P} (P - 1 - k) \neq 0 \qquad (4)$$

We prove that for every positive integer *k* there exist infinitely many primes P such that each of $P + j(j+1)$ is prime.

We have the asymptotic formula [1, 2, 3]

$$\pi_{k+1}(N, 2) = \left|\{P \le N : P + j(j+1) = prime\}\right| \sim \frac{J_2(\omega)\omega^k}{\phi^{k+1}(\omega)} \frac{N}{\log^{k+1} N}, \qquad (5)$$

where $\phi(\omega) = \prod_P (P-1)$.

Note Let $P = 11$, $11 + j(j+1)(j = 1, \cdots, 9)$ are all prime.

Let $P = 41$, $41 + j(j+1)(j = 1, \cdots, 39)$ are all prime.

Example 1. Let $k = 1, P, P + 2$, twin primes theorem.

From (4) we have

$$J_2(\omega)=\prod_{3\le P}(P-2)\ne 0 \quad (6)$$

We prove twin primes theorem. There exist infinitely many primes P such that $P+2$ is prime.
From (5) we have the best asymptotic formula

$$\pi_2(N,2)\sim 2\prod_{3\le P}\left(1-\frac{1}{(P-1)^2}\right)\frac{N}{\log^2 N} \quad (7)$$

Exampe 2. Let $k=2, P, P+2, P+6$.

From (4) we have

$$J_2(\omega)=\prod_{5\le P}(P-3)\ne 0 \quad (8)$$

We prove that there exist intinitely many primes P such that $P+2$ and $P+6$ are all prime.
From (5) we have the best asymptotic formula

$$\pi_3(N,2)\sim \frac{9}{2}\prod_{5\le P}\frac{P^2(P-3)}{(P-1)^3}\frac{N}{\log^3 N} \quad (9)$$

Example 3. Let $k=6, P, P+j(j+1)(j=1,\cdots,6)$

From (4) we have

$$J_2(\omega)=30\prod_{13\le P}(P-7)\ne 0 \quad (10)$$

We prove that there exist infinitely many primes P such that each of $P+j(j+1)$ is prime.
From (5) we have the best asymptotic formula

$$\pi_7(N,2)\sim \frac{1}{16}\left(\frac{231}{48}\right)^6\prod_{13\le P}\frac{(P-7)P^6}{(P-1)^7}\frac{N}{\log^7 N} \quad (11)$$

The author takes a day to write this paper.

References

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2. Chun-Xuan Jiang, The Hardy-Littlewood prime *k*-tuple conjecture is false. http:// www.wbabin.net/math/xuan77.pdf. This conjecture is generally believed to be true, but has not been proven (Odlyzko et al. 1999).
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Remark. Cramér's random model of prime theory is false.

Example. Assming that the events " P is prime" and " $P+2$ and $P+4$ are primes" are independent, we conclude that $P, P+2$ and $P+4$ are simultaneously prime with probability about $1/\log^3 N$. There are about $N/\log^3 N$ 3-tuple prime less than N. Letting $N\to\infty$ we obtain the 3-tuple conjecture which is false.

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