New prime K-tuple theorems(1) $P_1, P_2, P_1 + jP_2 + j (j = 1, \cdots, k)$ and $P_1, P_2, P_1 + jP_2 - j (j = 1, \cdots, k)$

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Abstract: Using Jinag funciton we prove that there exist infinitely many primes P_1 and P_2 such that each of $P_1 + jP_2 + j$ is prime and there exist infinitely many primes P_1 and P_2 such that each of $P_1 + jP_2 - j$ is prime.

[Chun-Xuan, Jiang. **New prime K -tuple theorems(1) $P_1, P_2, P_1 + jP_2 + j (j = 1, \cdots, k)$ and $P_1, P_2, P_1 + jP_2 - j (j = 1, \cdots, k)$**. *Academ Arena* 2016;8(2s): 1-2]. (ISSN 1553-992X). http://www.sciencepub.net/academia. 1. doi:10.7537/marsaaj0802s1601.

Keywords: new; prime; *k*-tuple; theorem; Jiang Chunxuan; mathematics; science; number; function

Theorem 1.

$$P_1, P_2, P_1 + jP_2 + j (j = 1, \cdots, k) \quad (1)$$

There exist infinitely many primes P_1 and P_2 such that each of $P_1 + jP_2 + j$ is prime.

Proof. We have Jiang function [1, 2]

$$J_3(\omega) = \prod_P [(P-1)^2 - \chi(P)] \quad (2)$$

where $\omega = \prod_P P$,

$\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k} (q_1 + jq_2 + j) \equiv 0 \pmod P, \quad q_i = 1, \cdots, P-1, i = 1, 2. \quad (3)$$

From (3) we have

If $k < P$ then $\chi(P) = k(P-2)$. If $P \le k$ then $\chi(P) = (P-1)(P-2)$.

From (3) and (2) we have

$$J_3(\omega) = \prod_{P=3}^{P \le k} (P-1) \prod_{k<P} [(P-1)^2 - k(P-2)] \ne 0 \quad (4)$$

For any positive integer k there exist inifinitely many primes P_1 and P_2 such that each of $P_1 + jP_2 + j$ is prime.

We have asymptotic formula [1, 2]

$$\pi_{k+1}(N, 3) = \left| \{ P_1, P_2 \le N : P_1 + jP_2 + j = prime \} \right| \sim \frac{J_3(\omega)\omega^k}{\phi^{k+2}(\omega)} \frac{N^2}{\log^{k+2} N}, \quad (5)$$

where $\phi(\omega) = \prod_P (P-1)$.

Example 1.

$$P_1, P_2, P_1 + P_2 + 1 \quad (6)$$

From (4) we have

$$J_3(\omega) = \prod_{3 \le P} [(P-1)^2 - (P-2)] \ne 0 \quad (7)$$

From (5) we have

$$\pi_2(N,3) \sim 2 \prod_{3 \le P} \left(1 + \frac{1}{(P-1)^3}\right) \frac{N^2}{\log^3 N} \quad (8)$$

Example 2.

$$P_1, P_2, P_1 + P_2 + 1, P_1 + 2P_2 + 2 \quad (9)$$

From (4) we have

$$J_3(\omega) = \prod_{3 \le P} [(P-1)^2 - 2(P-2)] \ne 0 \quad (10)$$

From (5) we have

$$\pi_3(N,3) \sim \frac{J_3(\omega)\omega^2}{\phi^4(\omega)} \frac{N^2}{\log^4 N} \quad (11)$$

Example 3.

$$P_1, P_2, P_1 + jP_2 + j (j = 1, \cdots, 8) \quad (12)$$

From (4) we have

$$J_3(\omega) = 48 \prod_{11 \le P} [(P-1)^2 - 8(P-2)] \ne 0 \quad (13)$$

From (5) we have

$$\pi_9(N,3) \sim \frac{J_3(\omega)\omega^8}{\phi^{10}(\omega)} \frac{N^2}{\log^{10} N} \quad (14)$$

Theorem 2.

$$P_1, P_2, P_1 + jP_2 - j (j = 1, \cdots, k) \quad (15)$$

we have Jiang function

$$J_3(\omega) = \prod_{P=3}^{P \le k} (P-1) \prod_{k < P} [(P-1)^2 - k(P-2)] \ne 0 \quad (16)$$

For any positive integer k there exist infinitely many primes P_1 and P_2 such that each of $P_1 + jP_2 - j$ is prime.

we have asymptotic formula

$$\pi_{k+1}(N,3) = \left|\{P_1, P_2 \le N : P_1 + jP_2 - j = prime\}\right| \sim \frac{J_3(\omega)\omega^k}{\phi^{k+2}(\omega)} \frac{N^2}{\log^{k+2} N} \quad (17)$$

References

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4/27/2016