

New prime K-tuple theorem (5)
 $P_1, P_2, jP_1 + (j+1)P_2 (j=1, \dots, k)$

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Abstract: Using Jiang function we prove that for every positive integer k there exist infinitely many primes P_1 and P_2 such that each of $jP_1 + (j+1)P_2$ is prime.

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Theorem

$$P_1, P_2, jP_1 + (j+1)P_2 (j=1, \dots, k) \quad (1)$$

For every positive integer k there exist infinitely many primes P_1 and P_2 such that each of $jP_1 + (j+1)P_2$ is prime.

Proof. We have Jiang function [1, 2]

$$J_3(\omega) = \prod_P [(P-1)^2 - \chi(P)] \quad (2)$$

where $\omega = \prod_P P$,

$\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^k [jq_1 + (j+1)q_2] \equiv 0 \pmod{P} \quad (3)$$

$$q_i = 1, \dots, P-1, i=1, 2.$$

From (3) we have

If $P \leq k+1$ then $\chi(P) = (P-1)(P-2)$, if $k+1 < P$ then $\chi(P) = k(P-1)$.

From (3) and (2) we have

$$J_3(\omega) = \prod_{P=3}^{P \leq k+1} (P-1) \prod_{k+1 < P} [(P-1)^2 - k(P-1)] \neq 0 \quad (4)$$

We prove that for every positive integer k there exist infinitely many primes P_1 and P_2 such that each of $jP_1 + (j+1)P_2$ is prime.

We have the best asymptotic formula [1, 2]

$$\pi_{k+1}(N, 3) = \left| \{P_1, P_2 \leq N : jP_1 + (j+1)P_2 = \text{prime}\} \right| \sim \frac{J_3(\omega)\omega^k}{\phi^{k+2}(\omega)} \frac{N^2}{\log^{k+2}}, \quad (5)$$

where $\phi(\omega) = \prod_P (P-1)$.

References

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