New prime K-tuple theorem (3)
$$P, jP + j + 1(j = 1, \cdots, k)$$

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Abstract: Using Jiang function we prove that for every positive integer k there exist infinitely many primes P such that each of $jP + j + 1$ is prime.

Keywords: new; prime; function; number

Theorem
$$P, jP + j + 1(j = 1, \cdots, k).$$
(1)

For every positive integer k there exist infinitely many primes P such that each of $jP + j + 1$ is prime.
Proof. We have Jiang function [1, 2]
$$J_2(\omega) = \prod_P (P - 1 - \chi(P)),$$
(2)

where $\omega = \prod_P P$,

$\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k}(jq + j + 1) \equiv 0 \pmod{P},$$
(3)

where $q = 1, \cdots, P - 1$.
From (3) we have

If $P \le k + 1$ then $\chi(P) = P - 2$, if $k + 1 < P$ then $\chi(P) = k$.
From (3) and (2) we have
$$J_2(\omega) = \prod_{k+1 < P}(P - k - 1) \ne 0.$$
(4)

We prove that for every positive integer k there exist infinitely many primes P such that each of $jP + j + 1$ is prime.
We have the best asymptotic formula [1, 2]

$$\pi_{k+1}(N, 2) = \left|\left\{P \le N : jP + j + 1 = prime\right\}\right| \sim \frac{J_2(\omega)\omega^k}{\phi^{k+1}(\omega)} \frac{N}{\log^{k+1} N}.$$
(5)

The author takes a day to write this paper.

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