

**New prime k-tuple theorem (2)**

$$P, P + j(j+1)(j = 1, \dots, k)$$

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**Abstract:** Using Jiang function we prove that for every positive integer  $k$  there exist infinitely many primes  $P$  such that each of  $P + j(j+1)$  is prime.

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**Theorem.**

$$P, P + j(j+1)(j = 1, \dots, k) \quad (1)$$

For every positive integer  $k$  there exist infinitely many primes  $P$  such that each of  $P + j(j+1)$  is prime.

**Proof.** We have Jiang function [1, 2, 3]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \quad (2)$$

$$\text{where } \omega = \prod_P P$$

$\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^k [q + j(j+1)] \equiv 0 \pmod{P} \quad (3)$$

where  $q = 1, \dots, P-1$ .

From (3) we have

$$\text{If } P < 2k \text{ then } \chi(P) = \frac{P-1}{2}, \text{ If } 2k < P \text{ then } \chi(P) = k.$$

From (3) and (2) we have.

$$J_2(\omega) = \prod_{P=3}^{P < 2k} \frac{P-1}{2} \prod_{2k < P} (P-1-k) \neq 0 \quad (4)$$

We prove that for every positive integer  $k$  there exist infinitely many primes  $P$  such that each of  $P + j(j+1)$  is prime.

We have the asymptotic formula [1, 2, 3]

$$\pi_{k+1}(N, 2) = \left| \{P \leq N : P + j(j+1) = \text{prime}\} \right| \sim \frac{J_2(\omega)\omega^k}{\phi^{k+1}(\omega) \log^{k+1} N} \quad (5)$$

$$\text{where } \phi(\omega) = \prod_P (P-1)$$

Note Let  $P = 11, 11 + j(j+1)(j = 1, \dots, 9)$  are all prime.

Let  $P = 41, 41 + j(j+1)(j = 1, \dots, 39)$  are all prime.

**Example 1.** Let  $k = 1, P, P + 2$ , twin primes theorem.

From (4) we have

$$J_2(\omega) = \prod_{3 \leq P} (P-2) \neq 0 \quad (6)$$

We prove twin primes theorem. There exist infinitely many primes  $P$  such that  $P + 2$  is prime. From (5) we have the best asymptotic formula

$$\pi_2(N, 2) \sim 2 \prod_{3 \leq P} \left(1 - \frac{1}{(P-1)^2}\right) \frac{N}{\log^2 N} \quad (7)$$

**Example 2.** Let  $k = 2, P, P + 2, P + 6$

From (4) we have

$$J_2(\omega) = \prod_{5 \leq P} (P-3) \neq 0 \quad (8)$$

We prove that there exist infinitely many primes  $P$  such that  $P + 2$  and  $P + 6$  are all prime. From (5) we have the best asymptotic formula

$$\pi_3(N, 2) \sim \frac{9}{2} \prod_{5 \leq P} \frac{P^2(P-3)}{(P-1)^3} \frac{N}{\log^3 N} \quad (9)$$

**Example 3.** Let  $k = 6, P, P + j(j+1)(j = 1, \dots, 6)$

From (4) we have

$$J_2(\omega) = 30 \prod_{13 \leq P} (P-7) \neq 0 \quad (10)$$

We prove that there exist infinitely many primes  $P$  such that each of  $P + j(j+1)$  is prime. From (5) we have the best asymptotic formula

$$\pi_7(N, 2) \sim \frac{1}{16} \left(\frac{231}{48}\right)^6 \prod_{13 \leq P} \frac{(P-7)P^6}{(P-1)^7} \frac{N}{\log^7 N} \quad (11)$$

The author takes a day to write this paper.

## References

1. Chun-Xuan Jiang, Jiang's function  $J_{n+1}(\omega)$  in prime distribution. (<http://www.wbabin.net/math/xuan2.pdf>)(<http://vixra.org/pdf/0812.0004v2.pdf>).
2. Chun-Xuan Jiang, The Hardy-Littlewood prime  $k$ -tuple conjecture is false. <http://www.wbabin.net/math/xuan77.pdf>. This conjecture is generally believed to be true, but has not been proven (Odlyzko et al. 1999).
3. Chun-Xuan Jiang, New prime  $k$ -tuple theorem (1), <http://www.wbabin.net/math/xuan78.pdf> <http://wbabin.net/xuan.htm#chun-xuan>

Remark. Cramér's random model of prime theory is false.

Example. Assuming that the events “ $P$  is prime” and “ $P + 2$  and  $P + 4$  are primes” are independent, we conclude that  $P, P + 2$  and  $P + 4$  are simultaneously prime with probability about  $1/\log^3 N$ . There are about  $N/\log^3 N$  3-tuple prime less than  $N$ . Letting  $N \rightarrow \infty$  we obtain the 3-tuple conjecture which is false.

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