

**New prime  $K$ -tuple theorems(1)**

$$P_1, P_2, P_1 + jP_2 + j(j=1, \dots, k) \quad \text{and} \quad P_1, P_2, P_1 + jP_2 - j(j=1, \dots, k)$$

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**Abstract:** Using Jinag fuciton we prove that there exist infinitely many primes  $P_1$  and  $P_2$  such that each of  $P_1 + jP_2 + j$  is prime and there exist infinitely many primes  $P_1$  and  $P_2$  such that each of  $P_1 + jP_2 - j$  is prime.

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**Keywords:** new; prime; function; number**Theorem 1.**

$$P_1, P_2, P_1 + jP_2 + j(j=1, \dots, k) \tag{1}$$

There exist infinitely many primes  $P_1$  and  $P_2$  such that each of  $P_1 + jP_2 + j$  is prime.

**Proof.** We have Jiang function [1, 2]

$$J_3(\omega) = \prod_P [(P-1)^2 - \chi(P)] \tag{2}$$

where  $\omega = \prod_P P$ ,

$\chi(P)$  is the number of solutions of congruence

$$\prod_{j=1}^k (q_1 + jq_2 + j) \equiv 0 \pmod{P}, \quad q_i = 1, \dots, P-1, i=1, 2. \tag{3}$$

From (3) we have

If  $k < P$  then  $\chi(P) = k(P-2)$ . If  $P \leq k$  then  $\chi(P) = (P-1)(P-2)$ .

From (3) and (2) we have

$$J_3(\omega) = \prod_{P=3}^{P \leq k} (P-1) \prod_{k < P} [(P-1)^2 - k(P-2)] \neq 0 \tag{4}$$

For any positive integer  $k$  there exist infinitely many primes  $P_1$  and  $P_2$  such that each of  $P_1 + jP_2 + j$  is prime.

We have asymptotic formula [1, 2]

$$\pi_{k+1}(N, 3) = \left| \{P_1, P_2 \leq N : P_1 + jP_2 + j = \text{prime}\} \right| \sim \frac{J_3(\omega) \omega^k}{\phi^{k+2}(\omega)} \frac{N^2}{\log^{k+2} N}, \tag{5}$$

where  $\phi(\omega) = \prod_P (P-1)$ .

**Example 1.**

$$P_1, P_2, P_1 + P_2 + 1 \tag{6}$$

From (4) we have

$$J_3(\omega) = \prod_{3 \leq p} [(P-1)^2 - (P-2)] \neq 0 \quad (7)$$

From (5) we have

$$\pi_2(N, 3) \sim 2 \prod_{3 \leq p} \left( 1 + \frac{1}{(P-1)^3} \right) \frac{N^2}{\log^3 N} \quad (8)$$

**Example 2.**

$$P_1, P_2, P_1 + P_2 + 1, P_1 + 2P_2 + 2 \quad (9)$$

From (4) we have

$$J_3(\omega) = \prod_{3 \leq p} [(P-1)^2 - 2(P-2)] \neq 0 \quad (10)$$

From (5) we have

$$\pi_3(N, 3) \sim \frac{J_3(\omega)\omega^2}{\phi^4(\omega)} \frac{N^2}{\log^4 N} \quad (11)$$

**Example 3.**

$$P_1, P_2, P_1 + jP_2 + j(j=1, \dots, 8) \quad (12)$$

From (4) we have

$$J_3(\omega) = 48 \prod_{11 \leq p} [(P-1)^2 - 8(P-2)] \neq 0 \quad (13)$$

From (5) we have

$$\pi_9(N, 3) \sim \frac{J_3(\omega)\omega^8}{\phi^{10}(\omega)} \frac{N^2}{\log^{10} N} \quad (14)$$

**Theorem 2.**

$$P_1, P_2, P_1 + jP_2 - j(j=1, \dots, k) \quad (15)$$

we have Jiang function

$$J_3(\omega) = \prod_{\substack{P \leq k \\ P=3}} (P-1) \prod_{k < P} [(P-1)^2 - k(P-2)] \neq 0 \quad (16)$$

For any positive integer  $k$  there exist infinitely many primes  $P_1$  and  $P_2$  such that each of  $P_1 + jP_2 - j$  is prime.

we have asymptotic formula

$$\pi_{k+1}(N, 3) = \left| \{P_1, P_2 \leq N : P_1 + jP_2 - j = \text{prime}\} \right| \sim \frac{J_3(\omega)\omega^k}{\phi^{k+2}(\omega)} \frac{N^2}{\log^{k+2} N} \quad (17)$$

## References

1. Chun-Xuan Jiang, Jiang's function  $J_{n+1}(\omega)$  in prime distribution. ([http:// www. Wbabin. net/math/xuan2. pdf](http://www.Wbabin.net/math/xuan2.pdf)) (<http://vixra.org/pdf/0812.0004v2.pdf>)
2. Chun-Xuan Jiang, The Hardy-Littlewood prime  $k$ -tuple conjecture is false. <http://www.wbabin.net/math/xuan77.pdf>

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