The New Prime theorems（891）-（940）

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Abstract: Using Jiang function we are able to prove almost all prime problems in prime distribution. This is the Book proof. No great mathematicians study prime problems and prove Riemann hypothesis in AIM, CLAYMI, IAS, THES, MPIM, MSRI. In this paper using Jiang function $J_2(\omega)$ we prove that the new prime theorems (891)-（940) contain infinitely many prime solutions and no prime solutions. From (6) we are able to find the smallest solution $\pi_k(N_0,2)\geq 1$. This is the Book theorem.

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It will be another million years, at least, before we understand the primes.
Paul Erdos (1913-1996)

The New Prime theorem（891）

$$P, jP^{1702}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1702}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1702}+k-j(j=1,\cdots,k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \quad (2)$$

where $\omega=\prod_{P}P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1702}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \quad (3)$$

If $\chi(P)\leq P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1702}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jP^{1702}+k-j=prime\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1702)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,47$. From (2) and(3) we have

$$J_2(\omega)=0 \quad (7)$$

we prove that for $k=3,47$

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,47$.

From (2) and (3) we have

$$J_2(\omega)\neq 0 \quad (8)$$

We prove that for $k\neq 3,47$,

(1) contain infinitely many prime solutions

The New Prime theorem（892）

$$P, jP^{1704}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1704}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1704}+k-j(j=1,\cdots,k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1704}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1704}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1704}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1704)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \tag{6}$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5,7,13,853$. From (2) and(3) we have

$$J_2(\omega)=0 \tag{7}$$

we prove that for $k=3,5,7,13,853$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,5,7,13,853$.

From (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{8}$$

We prove that for $k\neq 3,5,7,13,853$,

(1) contain infinitely many prime solutions

The New Prime theorem（893）

$$P, jP^{1706}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1706}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1706}+k-j(j=1,\cdots,k-1). \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \tag{2}$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1706}+k-j\right]\equiv 0 \pmod{P}, q=1,\cdots,P-1 \tag{3}$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes

P such that each of $jp^{1706}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jP^{1706}+k-j=prime\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1706)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \tag{6}$$

where $\phi(\omega)=\prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3$. From (2) and(3) we have

$$J_2(\omega)=0 \tag{7}$$

we prove that for $k=3$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3$.

From (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{8}$$

We prove that for $k\neq 3$,

(1) contain infinitely many prime solutions

The New Prime theorem（894）

$$P, jP^{1708}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1708}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1708}+k-j(j=1,\cdots,k-1). \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \tag{2}$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1708}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \tag{3}$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1708}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \qquad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\leq N: jP^{1708}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1708)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \qquad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\geq 1$.

Example 1. Let $k=3,5,29,1709$. From (2) and(3) we have

$$J_2(\omega)=0 \qquad (7)$$

we prove that for $k=3,5,29,1709$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,5,29,1709$.

From (2) and (3) we have

$$J_2(\omega)\neq 0 \qquad (8)$$

We prove that for $k\neq 3,5,29,1709$,

(1) contain infinitely many prime solutions

The New Prime theorem（895）

$$P, jP^{1710}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1710}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1710}+k-j(j=1,\cdots,k-1). \qquad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \qquad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1710}+k-j\right]\equiv 0 \pmod P), q=1,\cdots,P-1 \qquad (3)$$

If $\chi(P)\leq P-2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1710}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1710}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1710)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \tag{6}$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,7,11,19,31,191,571$. From (2) and(3) we have

$$J_2(\omega)=0 \tag{7}$$

we prove that for $k=3,7,11,19,31,191,571$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,7,11,19,31,191,571$.

From (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{8}$$

We prove that for $k\neq 3,7,11,19,31,191,571$,

(1) contain infinitely many prime solutions

The New Prime theorem（896）

$$P, jP^{1712}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1712}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1712}+k-j(j=1,\cdots,k-1). \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \tag{2}$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1712}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \tag{3}$$

If $\chi(P) \le P-2$ then from (2) and (3) we have

$$J_2(\omega) \ne 0$$ （4）

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1712}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P) = P-1$. Substituting it into (2) we have

$$J_2(\omega) = 0$$ （5）

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2) = \left|\left\{P \le N : jP^{1712}+k-j = prime\right\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(1712)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N}$$ （6）

where $\phi(\omega) = \prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2) \ge 1$.

Example 1. Let $k = 3,5,17,857$. From (2) and(3) we have

$$J_2(\omega) = 0$$ （7）

we prove that for $k = 3,5,17,857$,
(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k \ne 3,5,17,857$.

From (2) and (3) we have

$$J_2(\omega) \ne 0$$ （8）

We prove that for $k \ne 3,5,17,857$,
(1) contain infinitely many prime solutions

The New Prime theorem（897）

$$P, jP^{1714}+k-j (j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1714}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1714}+k-j (j=1,\cdots,k-1).$$ （1）

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_{P>2}[P-1-\chi(P)]$$ （2）

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1714}+k-j\right]\equiv 0 \pmod{P}, q=1,\cdots,P-1$$ （3）

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0$$ （4）

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1714}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0$$ （5）

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1714}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1714)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}$$ （6）

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3$. From (2) and(3) we have

$$J_2(\omega)=0$$ （7）

we prove that for $k=3$,
(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3$.
From (2) and (3) we have

$$J_2(\omega)\neq 0$$ （8）

We prove that for $k\neq 3$,
(1) contain infinitely many prime solutions

The New Prime theorem（898）

$$P, jP^{1716}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1716}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1716}+k-j(j=1,\cdots,k-1).$$ （1）

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]$$ （2）

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1716}+k-j\right]\equiv 0 \pmod P), q=1,\cdots,P-1 \tag{3}$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1716}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1716}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1716)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \tag{6}$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5,7,13,53,67,157,859$. From (2) and(3) we have

$$J_2(\omega)=0 \tag{7}$$

we prove that for $k=3,5,7,13,53,67,157,859$,
(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,5,7,13,53,67,157,859$.
From (2) and (3) we have

$$J_2(\omega)\ne 0 \tag{8}$$

We prove that for $k\ne 3,5,7,13,53,67,157,859$,
(1) contain infinitely many prime solutions

The New Prime theorem（899）

$$P, jP^{1718}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1718}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1718}+k-j(j=1,\cdots,k-1). \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \qquad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1718}+k-j\right]\equiv 0 \pmod P), q=1,\cdots,P-1 \qquad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \qquad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1718}{}_{+}k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \qquad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1718}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1718)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \qquad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3$. From (2) and(3) we have

$$J_2(\omega)=0 \qquad (7)$$

we prove that for $k=3$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3$.

From (2) and (3) we have

$$J_2(\omega)\ne 0 \qquad (8)$$

We prove that for $k\ne 3$,

(1) contain infinitely many prime solutions

The New Prime theorem（900）

$$P, jP^{1720}+k-j(j=1,\cdots,k-1)$$

Chun-Xuan Jiang
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Abstract

Using Jiang function we prove that $jP^{1720}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1720}+k-j(j=1,\cdots,k-1). \qquad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \tag{2}$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1720}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \tag{3}$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1720}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1720}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1720)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \tag{6}$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5,11,41,431,1721$
. From (2) and(3) we have

$$J_2(\omega)=0 \tag{7}$$

we prove that for $k=3,5,11,41,431,1721$,
(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,5,11,41,431,1721$.
From (2) and (3) we have

$$J_2(\omega)\ne 0 \tag{8}$$

We prove that for $k\ne 3,5,11,41,431,1721$,
(1) contain infinitely many prime solutions

The New Prime theorem（901）

$$P, jP^{1722}+k-j(j=1,\cdots,k-1)$$

Chun-Xuan Jiang
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Abstract

Using Jiang function we prove that $jP^{1722}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1722}+k-j(j=1,\cdots,k-1). \qquad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \qquad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1722}+k-j\right]\equiv 0 \pmod P), q=1,\cdots,P-1 \qquad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \qquad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1722}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \qquad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1722}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1722)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \qquad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,7,43,83,1723$. From (2) and(3) we have

$$J_2(\omega)=0 \qquad (7)$$

we prove that for $k=3,7,43,83,1723$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,7,43,83,1723$.

From (2) and (3) we have

$$J_2(\omega)\ne 0 \qquad (8)$$

We prove that for $k\ne 3,7,43,83,1723$,

(1) contain infinitely many prime solutions

The New Prime theorem（902）

$$P, jP^{1724}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1724}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1724}+k-j(j=1,\cdots,k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1724}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1724}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1724}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1724)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5,863$. From (2) and(3) we have

$$J_2(\omega)=0 \quad (7)$$

we prove that for $k=3,5,863$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,5,863$.

From (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (8)$$

We prove that for $k\ne 3,5,863$,

(1) contain infinitely many prime solutions

The New Prime theorem（903）

$$P, jP^{1726}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1726}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1726}+k-j(j=1,\cdots,k-1). \qquad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \qquad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1726}+k-j\right]\equiv 0 \pmod P), q=1,\cdots,P-1 \qquad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \qquad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1726}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \qquad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1726}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1726)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \qquad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3$. From (2) and(3) we have

$$J_2(\omega)=0 \qquad (7)$$

we prove that for $k=3$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3$.

From (2) and (3) we have

$$J_2(\omega)\ne 0 \qquad (8)$$

We prove that for $k\ne 3$,

(1) contain infinitely many prime solutions

The New Prime theorem（904）

$$P, jP^{1728}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1728}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1728}+k-j(j=1,\cdots,k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1728}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1728}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1728}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1728)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5,7,13,17,19,37,73,97,109,433$. From (2) and(3) we have

$$J_2(\omega)=0 \quad (7)$$

we prove that for $k=3,5,7,13,17,19,37,73,97,109,433$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,5,7,13,17,19,37,73,97,109,433$.

From (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (8)$$

We prove that for $k\ne 3,5,7,13,17,19,37,73,97,109,433$,

(1) contain infinitely many prime solutions

The New Prime theorem（905）

$$P, jP^{1730}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1730}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1730}+k-j(j=1,\cdots,k-1). \qquad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \qquad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1730}+k-j\right]\equiv 0 \pmod P), q=1,\cdots,P-1 \qquad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \qquad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1730}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \qquad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1730}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1730)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \qquad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,11,347$. From (2) and(3) we have

$$J_2(\omega)=0 \qquad (7)$$

we prove that for $k=3,11,347$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,11,347$.

From (2) and (3) we have

$$J_2(\omega)\ne 0 \qquad (8)$$

We prove that for $k\ne 3,11,347$,

(1) contain infinitely many prime solutions

The New Prime theorem（906）

$P, jP^{1732}+k-j(j=1,\cdots,k-1)$

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Abstract

Using Jiang function we prove that $jP^{1732}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1732}+k-j(j=1,\cdots,k-1). \qquad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \qquad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1732}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \qquad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \qquad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1732}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \qquad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1732}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1732)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \qquad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5,1733$. From (2) and(3) we have

$$J_2(\omega)=0 \qquad (7)$$

we prove that for $k=3,5,1733$,
(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,5,1733$.
From (2) and (3) we have

$$J_2(\omega)\ne 0 \qquad (8)$$

We prove that for $k\ne 3,5,1733$,
(1) contain infinitely many prime solutions

The New Prime theorem（907）

$$P, jP^{1734}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1734}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1734}+k-j(j=1,\cdots,k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1734}+k-j\right]\equiv 0 \ (\bmod P), q=1,\cdots,P-1 \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1734}+k-j$ is a prime.

Using Fermat’s little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1734}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1734)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,7,103$. From (2) and(3) we have

$$J_2(\omega)=0 \quad (7)$$

we prove that for $k=3,7,103$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,7,103$.

From (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (8)$$

We prove that for $k\ne 3,7,103$,

(1) contain infinitely many prime solutions

The New Prime theorem（908）

$$P, jP^{1736}+k-j (j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1736}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1736}+k-j (j=1,\cdots,k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1736}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1736}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1736}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1736)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5,29$. From (2) and(3) we have

$$J_2(\omega)=0 \quad (7)$$

we prove that for $k=3,5,29$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,5,29$.

From (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (8)$$

We prove that for $k \neq 3,5,29$,
(1) contain infinitely many prime solutions

The New Prime theorem（909）

$$P, jP^{1738}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1738}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1738}+k-j(j=1,\cdots,k-1). \qquad (1)$$

contain infinitely many prime solutions and no prime solutions.
Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \qquad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1738}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \qquad (3)$$

If $\chi(P)\leq P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \qquad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1738}+k-j$ is a prime.

Using Fermat’s little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \qquad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\leq N: jP^{1738}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1738)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \qquad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\geq 1$.

Example 1. Let $k=3,23$. From (2) and(3) we have

$$J_2(\omega)=0 \qquad (7)$$

we prove that for $k=3,23$,
(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,23$.
From (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{8}$$

We prove that for $k\neq 3,23$,

(1) contain infinitely many prime solutions

The New Prime theorem（910）

$$P, jP^{1740}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1740}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1740}+k-j(j=1,\cdots,k-1). \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \tag{2}$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1740}+k-j\right]\equiv 0 \pmod P), q=1,\cdots,P-1 \tag{3}$$

If $\chi(P)\leq P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1740}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\leq N: jP^{1740}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1740)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \tag{6}$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\geq 1$.

Example 1. Let $k=3,5,7,11,13,31,59,61,349,1741$. From (2) and(3) we have

$$J_2(\omega)=0 \tag{7}$$

we prove that for $k=3,5,7,11,13,31,59,61,349,1741$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3,5,7,11,13,31,59,61,349,1741$.

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for $k \neq 3,5,7,11,13,31,59,61,349,1741$,

(1) contain infinitely many prime solutions

The New Prime theorem（911）

$$P, jP^{1742}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1742}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1742}+k-j(j=1,\cdots,k-1). \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \tag{2}$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1742}+k-j\right]\equiv 0 \pmod{P}, q=1,\cdots,P-1 \tag{3}$$

If $\chi(P)\leq P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1742}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\leq N: jP^{1742}+k-j=prime\right\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(1742)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \tag{6}$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\geq 1$.

Example 1. Let $k=3$. From (2) and(3) we have

$$J_2(\omega)=0 \tag{7}$$

we prove that for $k=3$,
(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3$.
From (2) and (3) we have

$$J_2(\omega) \neq 0 \qquad (8)$$

We prove that for $k \neq 3$,
(1) contain infinitely many prime solutions

The New Prime theorem（912）

$$P, jP^{1744}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1744}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1744}+k-j(j=1,\cdots,k-1). \qquad (1)$$

contain infinitely many prime solutions and no prime solutions.
Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \qquad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1744}+k-j\right] \equiv 0 \pmod P, q=1,\cdots,P-1 \qquad (3)$$

If $\chi(P) \leq P-2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \qquad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1744}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \qquad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P \leq N : jP^{1744}+k-j=prime\right\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(1744)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \qquad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2) \geq 1$.

Example 1. Let $k=3,5,17$. From (2) and(3) we have

$$J_2(\omega)=0 \tag{7}$$

we prove that for $k=3,5,17$,
(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,5,17$.
From (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{8}$$

We prove that for $k\neq 3,5,17$,
(1) contain infinitely many prime solutions

The New Prime theorem（913）

$$P, jP^{1746}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1746}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1746}+k-j(j=1,\cdots,k-1). \tag{1}$$

contain infinitely many prime solutions and no prime solutions.
Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \tag{2}$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1746}+k-j\right]\equiv 0 \pmod P), q=1,\cdots,P-1 \tag{3}$$

If $\chi(P)\leq P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1746}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\leq N: jP^{1746}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1746)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \tag{6}$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\geq 1$.

Example 1. Let $k=3,7,19,1747$. From (2) and(3) we have

$$J_2(\omega)=0 \tag{7}$$

we prove that for $k=3,7,19,1747$,
(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,7,19,1747$.
From (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{8}$$

We prove that for $k\neq 3,7,19,1747$,
(1) contain infinitely many prime solutions

The New Prime theorem（914）

$$P, jP^{1748}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1748}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1748}+k-j(j=1,\cdots,k-1). \tag{1}$$

contain infinitely many prime solutions and no prime solutions.
Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \tag{2}$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1748}+k-j\right]\equiv 0 \pmod P), q=1,\cdots,P-1 \tag{3}$$

If $\chi(P)\leq P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1748}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\leq N: jP^{1748}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1748)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \tag{6}$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5,47$. From (2) and(3) we have

$$J_2(\omega)=0 \qquad (7)$$

we prove that for $k=3,5,47$,
(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,5,47$.
From (2) and (3) we have

$$J_2(\omega)\ne 0 \qquad (8)$$

We prove that for $k\ne 3,5,47$,
(1) contain infinitely many prime solutions

The New Prime theorem（915）

$$P, jP^{1750}+k-j (j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1750}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1750}+k-j (j=1,\cdots,k-1). \qquad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \qquad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1750}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \qquad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \qquad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1750}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \qquad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1750}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1750)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \qquad (6)$$

where $\phi(\omega)=\prod_{P}(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\geq 1$.

Example 1. Let $k=3,11,71,251$. From (2) and(3) we have

$$J_2(\omega)=0 \tag{7}$$

we prove that for $k=3,11,71,251$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,11,71,251$.

From (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{8}$$

We prove that for $k\neq 3,11,71,251$,

(1) contain infinitely many prime solutions

The New Prime theorem（916）

$$P, jP^{1752}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1752}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1752}+k-j(j=1,\cdots,k-1). \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \tag{2}$$

where $\omega=\prod_{P}P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1752}+k-j\right]\equiv 0 \pmod{P}, q=1,\cdots,P-1 \tag{3}$$

If $\chi(P)\leq P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1752}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1752}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1752)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \qquad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5,7,13,293,1753$. From (2) and(3) we have

$$J_2(\omega)=0 \qquad (7)$$

we prove that for $k=3,5,7,13,293,1753$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,5,7,13,293,1753$.

From (2) and (3) we have

$$J_2(\omega)\ne 0 \qquad (8)$$

We prove that for $k\ne 3,5,7,13,293,1753$,

(1) contain infinitely many prime solutions

The New Prime theorem（917）

$$P, jP^{1754}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1754}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1754}+k-j(j=1,\cdots,k-1). \qquad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \qquad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1754}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \qquad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \qquad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1754}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \qquad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\{P\le N: jP^{1754}+k-j=prime\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1754)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \tag{6}$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3$. From (2) and(3) we have

$$J_2(\omega)=0 \tag{7}$$

we prove that for $k=3$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3$.

From (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{8}$$

We prove that for $k\neq 3$,

(1) contain infinitely many prime solutions

The New Prime theorem（918）

$$P, jP^{1756}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1756}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1756}+k-j(j=1,\cdots,k-1). \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \tag{2}$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1756}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \tag{3}$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1756}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1756}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1756)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \tag{6}$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5$. From (2) and(3) we have

$$J_2(\omega)=0 \tag{7}$$

we prove that for $k=3,5$,
(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,5$.
From (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{8}$$

We prove that for $k\neq 3,5$,
(1) contain infinitely many prime solutions

The New Prime theorem（919）

$$P, jP^{1758}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1758}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1758}+k-j(j=1,\cdots,k-1). \tag{1}$$

contain infinitely many prime solutions and no prime solutions.
Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \tag{2}$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1758}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \tag{3}$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1758}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1758}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1758)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \tag{6}$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,7,1759$. From (2) and(3) we have

$$J_2(\omega)=0 \tag{7}$$

we prove that for $k=3,7,1759$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,7,1759$.

From (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{8}$$

We prove that for $k\neq 3,7,1759$,

(1) contain infinitely many prime solutions

The New Prime theorem（920）

$$P, jP^{1760}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1760}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1760}+k-j(j=1,\cdots,k-1). \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \tag{2}$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1760}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \tag{3}$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes

P such that each of $jp^{1760}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1760}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1760)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \tag{6}$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5,11,17,23,41,89,353,881$. From (2) and(3) we have

$$J_2(\omega)=0 \tag{7}$$

we prove that for $k=3,5,11,17,23,41,89,353,881$,
(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,5,11,17,23,41,89,353,881$.

From (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{8}$$

We prove that for $k\neq 3,5,11,17,23,41,89,353,881$,
(1) contain infinitely many prime solutions

The New Prime theorem（921）

$$P, jP^{1762}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1762}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1762}+k-j(j=1,\cdots,k-1). \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \tag{2}$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1762}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \tag{3}$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1762}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1762}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1762)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \tag{6}$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3$. From (2) and(3) we have

$$J_2(\omega)=0 \tag{7}$$

we prove that for $k=3$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3$.

From (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{8}$$

We prove that for $k\neq 3$,

(1) contain infinitely many prime solutions

The New Prime theorem（922）

$$P, jP^{1764}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1764}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1764}+k-j(j=1,\cdots,k-1). \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \tag{2}$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1764}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \tag{3}$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1764}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1764}+k-j=prime\right\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(1764)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \tag{6}$$

where $\phi(\omega)=\prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5,7,13,19,29,37,43,127,883$. From (2) and(3) we have

$$J_2(\omega)=0 \tag{7}$$

we prove that for $k=3,5,7,13,19,29,37,43,127,883$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,5,7,13,19,29,37,43,127,883$.

From (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{8}$$

We prove that for $k\neq 3,5,7,13,19,29,37,43,127,883$,

(1) contain infinitely many prime solutions

The New Prime theorem（923）

$$P, jP^{1766}+k-j (j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1766}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1766}+k-j (j=1,\cdots,k-1). \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \tag{2}$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1766}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \tag{3}$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1766}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1766}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1766)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \tag{6}$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3$. From (2) and(3) we have

$$J_2(\omega)=0 \tag{7}$$

we prove that for $k=3$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3$.

From (2) and (3) we have

$$J_2(\omega)\ne 0 \tag{8}$$

We prove that for $k\ne 3$,

(1) contain infinitely many prime solutions

The New Prime theorem（924）

$$P, jP^{1768}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1768}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1768}+k-j(j=1,\cdots,k-1). \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1768}+k-j \right] \equiv 0 \pmod P, q=1,\cdots,P-1 \tag{3}$$

If $\chi(P) \le P-2$ then from (2) and (3) we have

$$J_2(\omega) \ne 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1768}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P \le N : jP^{1768}+k-j = prime\right\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(1768)^{k-1}\phi^k(\omega)} \frac{N}{\log^k N} \tag{6}$$

where $\phi(\omega)=\prod_P (P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2) \ge 1$.

Example 1. Let $k=3,5,53,127,443$. From (2) and(3) we have

$$J_2(\omega)=0 \tag{7}$$

we prove that for $k=3,5,53,127,443$,
(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k \ne 3,5,53,127,443$.
From (2) and (3) we have

$$J_2(\omega) \ne 0 \tag{8}$$

We prove that for $k \ne 3,5,53,127,443$,
(1) contain infinitely many prime solutions

The New Prime theorem（925）

$$P, jP^{1770}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1770}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1770}+k-j(j=1,\cdots,k-1). \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \tag{2}$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1770}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \tag{3}$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1770}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1770}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1770)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \tag{6}$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,7,11,31$. From (2) and(3) we have

$$J_2(\omega)=0 \tag{7}$$

we prove that for $k=3,7,11,31$,
(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,7,11,31$.
From (2) and (3) we have

$$J_2(\omega)\ne 0 \tag{8}$$

We prove that for $k\ne 3,7,11,31$,
(1) contain infinitely many prime solutions

The New Prime theorem（926）

$$P, jP^{1772}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1772}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1772}+k-j(j=1,\cdots,k-1) .\tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)]\tag{2}$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1772}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1\tag{3}$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0\tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1772}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0\tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1772}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1772)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N}\tag{6}$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5,887$. From (2) and(3) we have

$$J_2(\omega)=0\tag{7}$$

we prove that for $k=3,5,887$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,5,887$.

From (2) and (3) we have

$$J_2(\omega)\ne 0\tag{8}$$

We prove that for $k\ne 3,5,887$,

(1) contain infinitely many prime solutions

The New Prime theorem（927）

$$P, jP^{1774}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1774}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1774}+k-j(j=1,\cdots,k-1). \qquad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \qquad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1774}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \qquad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \qquad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1774}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \qquad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1774}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1774)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \qquad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3$. From (2) and(3) we have

$$J_2(\omega)=0 \qquad (7)$$

we prove that for $k=3$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3$.

From (2) and (3) we have

$$J_2(\omega)\ne 0 \qquad (8)$$

We prove that for $k\ne 3$,

(1) contain infinitely many prime solutions

The New Prime theorem（928）

$$P, jP^{1776}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1776}+k-j$ contain infinitely many prime solutions and no prime

solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1776}+k-j(j=1,\cdots,k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1776}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1776}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1776}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1776)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5,7,13,17,149,593,1777$. From (2) and(3) we have

$$J_2(\omega)=0 \quad (7)$$

we prove that for $k=3,5,7,13,17,149,593,1777$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,5,7,13,17,149,593,1777$.

From (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (8)$$

We prove that for $k\ne 3,5,7,13,17,149,593,1777$,

(1) contain infinitely many prime solutions

The New Prime theorem（929）

$$P, jP^{1778}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1778}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1778}+k-j(j=1,\cdots,k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1778}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1778}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1778}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1778)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3$. From (2) and(3) we have

$$J_2(\omega)=0 \quad (7)$$

we prove that for $k=3$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3$.

From (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (8)$$

We prove that for $k\ne 3$,

(1) contain infinitely many prime solutions

The New Prime theorem（930）

$$P, jP^{1780}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1780}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1780}+k-j(j=1,\cdots,k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1780}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1780}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1780}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1780)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5,11$. From (2) and(3) we have

$$J_2(\omega)=0 \quad (7)$$

we prove that for $k=3,5,11$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,5,11$.

From (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (8)$$

We prove that for $k\ne 3,5,11$,

(1) contain infinitely many prime solutions

The New Prime theorem（931）

$$P, jP^{1782}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1782}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1782}+k-j(j=1,\cdots,k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1782}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1782}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1782}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1782)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,7,19,23,67,163,199,1783$. From (2) and(3) we have

$$J_2(\omega)=0 \quad (7)$$

we prove that for $k=3,7,19,23,67,163,199,1783$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,7,19,23,67,163,199,1783$.

From (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (8)$$

We prove that for $k\ne 3,7,19,23,67,163,199,1783$,

(1) contain infinitely many prime solutions

The New Prime theorem（932）

$P, jP^{1784}+k-j(j=1,\cdots,k-1)$

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Abstract

Using Jiang function we prove that $jP^{1784}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1784}+k-j(j=1,\cdots,k-1). \quad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1784}+k-j\right]\equiv 0 \pmod P), q=1,\cdots,P-1 \quad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1784}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \quad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1784}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1784)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \quad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5$. From (2) and(3) we have

$$J_2(\omega)=0 \quad (7)$$

we prove that for $k=3,5$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,5$.

From (2) and (3) we have

$$J_2(\omega)\ne 0 \quad (8)$$

We prove that for $k\ne 3,5$,

(1) contain infinitely many prime solutions

The New Prime theorem（933）

$$P, jP^{1786}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1786}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1786}+k-j(j=1,\cdots,k-1). \qquad (1)$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \qquad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1786}+k-j\right]\equiv 0 \ (\bmod P), q=1,\cdots,P-1 \qquad (3)$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \qquad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1786}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \qquad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1786}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1786)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \qquad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,1787$. From (2) and(3) we have

$$J_2(\omega)=0 \qquad (7)$$

we prove that for $k=3,1787$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,1787$.

From (2) and (3) we have

$$J_2(\omega)\ne 0 \qquad (8)$$

We prove that for $k\ne 3,1787$,

(1) contain infinitely many prime solutions

The New Prime theorem（934）

$$P, jP^{1788}+k-j (j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1788}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1788}+k-j (j=1,\cdots,k-1). \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \tag{2}$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1788}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \tag{3}$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\ne 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1788}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\ne 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1788}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1788)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \tag{6}$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5,7,13,1789$. From (2) and(3) we have

$$J_2(\omega)=0 \tag{7}$$

we prove that for $k=3,5,7,13,1789$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\ne 3,5,7,13,1789$.

From (2) and (3) we have

$$J_2(\omega)\ne 0 \tag{8}$$

We prove that for $k \neq 3,5,7,13,1789$,
(1) contain infinitely many prime solutions

The New Prime theorem（935）

$$P, jP^{1790}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1790}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1790}+k-j(j=1,\cdots,k-1). \tag{1}$$

contain infinitely many prime solutions and no prime solutions.
Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \tag{2}$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1790}+k-j\right]\equiv 0 \pmod{P}, q=1,\cdots,P-1 \tag{3}$$

If $\chi(P)\leq P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1790}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\leq N: jP^{1790}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1790)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \tag{6}$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\geq 1$.

Example 1. Let $k=3,11,359$. From (2) and(3) we have

$$J_2(\omega)=0 \tag{7}$$

we prove that for $k=3,11,359$,
(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,11,359$.
From (2) and (3) we have

$$J_2(\omega)\neq 0 \qquad (8)$$

We prove that for $k\neq 3,11,359$,
(1) contain infinitely many prime solutions

The New Prime theorem（936）

$$P, jP^{1792}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1792}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1792}+k-j(j=1,\cdots,k-1). \qquad (1)$$

contain infinitely many prime solutions and no prime solutions.
Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \qquad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1792}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \qquad (3)$$

If $\chi(P)\leq P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \qquad (4)$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1792}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \qquad (5)$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\leq N: jP^{1792}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1792)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \qquad (6)$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\geq 1$.

Example 1. Let $k=3,5,17,29,113,257,449$. From (2) and(3) we have

$$J_2(\omega)=0 \qquad (7)$$

we prove that for $k=3,5,17,29,113,257,449$,
(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3,5,17,29,113,257,449$.

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for $k \neq 3,5,17,29,113,257,449$,

(1) contain infinitely many prime solutions

The New Prime theorem（937）

$$P, jP^{1794}+k-j(j=1,\cdots,k-1)$$

Chun-Xuan Jiang
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Abstract

Using Jiang function we prove that $jP^{1794}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1794}+k-j(j=1,\cdots,k-1). \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \tag{2}$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1794}+k-j\right] \equiv 0 \pmod{P}, q=1,\cdots,P-1 \tag{3}$$

If $\chi(P) \leq P-2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1794}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P \leq N : jP^{1794}+k-j=prime\right\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(1794)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \tag{6}$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2) \geq 1$.

Example 1. Let $k=3,7,47,79,139$. From (2) and(3) we have

$$J_2(\omega)=0 \tag{7}$$

we prove that for $k=3,7,47,79,139$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k \neq 3,7,47,79,139$.

From (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{8}$$

We prove that for $k \neq 3,7,47,79,139$,

(1) contain infinitely many prime solutions

The New Prime theorem（938）

$$P, jP^{1796}+k-j (j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1796}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1796}+k-j (j=1,\cdots,k-1). \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \tag{2}$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1796}+k-j\right] \equiv 0 \pmod P, q=1,\cdots,P-1 \tag{3}$$

If $\chi(P) \leq P-2$ then from (2) and (3) we have

$$J_2(\omega) \neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1796}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega) \neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P \leq N: jP^{1796}+k-j=prime\right\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{(1796)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \tag{6}$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2) \geq 1$.

Example 1. Let $k=3,5$. From (2) and(3) we have

$$J_2(\omega)=0 \tag{7}$$

we prove that for $k=3,5$,
(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,5$.
From (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{8}$$

We prove that for $k\neq 3,5$,
(1) contain infinitely many prime solutions

The New Prime theorem（939）

$$P, jP^{1798}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1798}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1798}+k-j(j=1,\cdots,k-1). \tag{1}$$

contain infinitely many prime solutions and no prime solutions.
Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \tag{2}$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1798}+k-j\right]\equiv 0 \pmod P), q=1,\cdots,P-1 \tag{3}$$

If $\chi(P)\leq P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1798}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\leq N: jP^{1798}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1798)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \tag{6}$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\geq 1$.

Example 1. Let $k=3,59$. From (2) and(3) we have

$$J_2(\omega)=0 \tag{7}$$

we prove that for $k=3,59$,

(1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,59$.

From (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{8}$$

We prove that for $k\neq 3,59$,

(1) contain infinitely many prime solutions

The New Prime theorem（940）

$$P, jP^{1800}+k-j(j=1,\cdots,k-1)$$

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Abstract

Using Jiang function we prove that $jP^{1800}+k-j$ contain infinitely many prime solutions and no prime solutions.

Theorem. Let k be a given odd prime.

$$P, jP^{1800}+k-j(j=1,\cdots,k-1). \tag{1}$$

contain infinitely many prime solutions and no prime solutions.

Proof. We have Jiang function [1,2]

$$J_2(\omega)=\prod_{P>2}[P-1-\chi(P)] \tag{2}$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1}\left[jq^{1800}+k-j\right]\equiv 0 \pmod P, q=1,\cdots,P-1 \tag{3}$$

If $\chi(P)\le P-2$ then from (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{4}$$

We prove that (1) contain infinitely many prime solutions that is for any k there are infinitely many primes P such that each of $jp^{1800}+k-j$ is a prime.

Using Fermat's little theorem from (3) we have $\chi(P)=P-1$. Substituting it into (2) we have

$$J_2(\omega)=0 \tag{5}$$

We prove that (1) contain no prime solutions [1,2]

If $J_2(\omega)\neq 0$ then we have asymptotic formula [1,2]

$$\pi_k(N,2)=\left|\left\{P\le N: jP^{1800}+k-j=prime\right\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{(1800)^{k-1}\phi^k(\omega)}\frac{N}{\log^k N} \tag{6}$$

where $\phi(\omega)=\prod_P(P-1)$.

From (6) we are able to find the smallest solution $\pi_k(N_0,2)\ge 1$.

Example 1. Let $k=3,5,7,11,13,19,31,37,41,61,101,151,181,1801$. From (2) and(3) we have

$$J_2(\omega)=0 \tag{7}$$

we prove that for $k=3,5,7,11,13,19,31,37,41,61,101,151,181,1801$, (1) contain no prime solutions. 1 is not a prime.

Example 2. Let $k\neq 3,5,7,11,13,19,31,37,41,61,101,151,181,1801$.

From (2) and (3) we have

$$J_2(\omega)\neq 0 \tag{8}$$

We prove that for $k\neq 3,5,7,11,13,19,31,37,41,61,101,151,181,1801$, (1) contain infinitely many prime solutions

Remark. The prime number theory is basically to count the Jiang function $J_{n+1}(\omega)$ and Jiang prime k-tuple singular series $\sigma(J)=\frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)}=\prod_P\left(1-\frac{1+\chi(P)}{P}\right)(1-\frac{1}{P})^{-k}$ [1,2], which can count the number of prime numbers. The prime distribution is not random. But Hardy-Littlewood prime k-tuple singular series $\sigma(H)=\prod_P\left(1-\frac{\nu(P)}{P}\right)(1-\frac{1}{P})^{-k}$ is false [3-17], which cannot count the number of prime numbers[3].

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Szemerédi’s theorem does not directly to the primes, because it cannot count the number of primes. Cramér’s random model cannot prove any prime problems. The probability of $1/\log N$ of being prime is false. Assuming that the events “P is prime”, “$P+2$ is prime” and “$P+4$ is prime” are independent, we conclude that P, $P+2$, $P+4$ are simultaneously prime with probability about $1/\log^3 N$. There are about $N/\log^3 N$ primes less than N. Letting $N\to\infty$ we obtain the prime conjecture, which is false. The tool of additive prime number theory is basically the Hardy-Littlewood prime tuples conjecture, but cannot prove and count any prime problems[6].

Mathematicians have tried in vain to discover some order in the sequence of prime numbers but we have every reason to believe that there are some mysteries which the human mind will never penetrate.

Leonhard Euler(1707-1783)

It will be another million years, at least, before we understand the primes.

Paul Erdos(1913-1996)

Jiang’s function $J_{n+1}(\omega)$ in prime distribution

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Dedicated to the 30-th anniversary of hadronic mechanics

Abstract: We define that prime equations

$$f_1(P_1,\cdots,P_n),\cdots,f_k(P_1,\cdots P_n)$$ （5）

are polynomials (with integer coefficients) irreducible over integers, where $P_1,\cdots,P_n$ are all prime. If Jiang’s function $J_{n+1}(\omega)=0$ then （5）has finite prime solutions. If $J_{n+1}(\omega)\neq 0$ then there are infinitely many primes $P_1,\cdots,P_n$ such that $f_1,\cdots f_k$ are primes. We obtain a unite prime formula in prime distribution

$$\pi_{k+1}(N,n+1)=\left|\{P_1,\cdots,P_n\le N: f_1,\cdots,f_k \text{ are } k \text{ primes}\}\right|$$
$$=\prod_{i=1}^{k}(\deg f_i)^{-1}\times\frac{J_{n+1}(\omega)\omega^k}{n!\phi^{k+n}(\omega)}\frac{N^n}{\log^{k+n}N}(1+o(1)).$$ （8）

Jiang’s function is accurate sieve function. Using Jiang’s function we prove about 600 prime theorems [6]. Jiang’s function provides proofs of the prime theorems which are simple enough to understand and accurate enough to be useful.

Mathematicians have tried in vain to discover some order in the sequence of prime numbers but we have every reason to believe that there are some mysteries which the human mind will never penetrate.

Leonhard Euler

It will be another million years, at least, before we understand the primes.

Paul Erdös

Suppose that Euler totient function

$$\phi(\omega)=\prod_{2\le P}(P-1)=\infty \quad \text{as} \quad \omega\to\infty,$$ （1）

where $\omega = \prod_{2 \le P} P$ is called primorial.

Suppose that $(\omega, h_i) = 1$, where $i = 1, \cdots, \phi(\omega)$. We have prime equations

$$P_1 = \omega n + 1, \cdots, P_{\phi(\omega)} = \omega n + h_{\phi(\omega)} \qquad (2)$$

where $n = 0, 1, 2, \cdots$.

（2）is called infinitely many prime equations (IMPE). Every equation has infinitely many prime solutions. We have

$$\pi_{h_i} = \sum_{\substack{P_i \le N \\ P_i \equiv h_i \pmod{\omega}}} 1 = \frac{\pi(N)}{\phi(\omega)}(1 + o(1)), \qquad (3)$$

where π_{h_i} denotes the number of primes $P_i \le N$ in $P_i = \omega n + h_i$ $n = 0, 1, 2, \cdots$, $\pi(N)$ the number of primes less than or equal to N.

We replace sets of prime numbers by IMPE. (2) is the fundamental tool for proving the prime theorems in prime distribution.

Let $\omega = 30$ and $\phi(30) = 8$. From (2) we have eight prime equations

$$P_1 = 30n + 1, \ P_2 = 30n + 7, \ P_3 = 30n + 11, \ P_4 = 30n + 13, \ P_5 = 30n + 17,$$
$$P_6 = 30n + 19, \ P_7 = 30n + 23, \ P_8 = 30n + 29, \ n = 0, 1, 2, \cdots \qquad (4)$$

Every equation has infinitely many prime solutions.

THEOREM. We define that prime equations

$$f_1(P_1, \cdots, P_n), \cdots, f_k(P_1, \cdots, P_n) \qquad (5)$$

are polynomials (with integer coefficients) irreducible over integers, where $P_1, \cdots, P_n$ are primes. If Jiang's function $J_{n+1}(\omega) = 0$ then (5) has finite prime solutions. If $J_{n+1}(\omega) \ne 0$ then there exist infinitely many primes $P_1, \cdots, P_n$ such that each f_k is a prime.

PROOF. Firstly, we have Jiang's function [1-11]

$$J_{n+1}(\omega) = \prod_{3 \le P} [(P-1)^n - \chi(P)], \qquad (6)$$

where $\chi(P)$ is called sieve constant and denotes the number of solutions for the following congruence

$$\prod_{i=1}^{k} f_i(q_1, \cdots, q_n) \equiv 0 \pmod{P}, \qquad (7)$$

where $q_1 = 1, \cdots, P-1, \cdots, q_n = 1, \cdots, P-1$.

$J_{n+1}(\omega)$ denotes the number of sets of $P_1, \cdots, P_n$ prime equations such that $f_1(P_1, \cdots, P_n), \cdots, f_k(P_1, \cdots, P_n)$ are prime equations. If $J_{n+1}(\omega) = 0$ then (5) has finite prime solutions. If $J_{n+1}(\omega) \ne 0$ using $\chi(P)$ we sift out from (2) prime equations which can not be represented $P_1, \cdots, P_n$, then residual prime equations of (2) are $P_1, \cdots, P_n$ prime equations such that $f_1(P_1, \cdots, P_n), \cdots,$ $f_k(P_1, \cdots, P_n)$ are prime equations. Therefore we prove that there exist infinitely many primes $P_1, \cdots, P_n$ such that $f_1(P_1, \cdots, P_n), \cdots,$ $f_k(P_1, \cdots, P_n)$ are primes.

Secondly, we have the best asymptotic formula [2,3,4,6]

$$\pi_{k+1}(N, n+1) = \left| \{P_1, \cdots, P_n \le N : f_1, \cdots, f_k \text{ are } k \text{ primes}\} \right|$$

$$=\prod_{i=1}^{k}(\deg f_i)^{-1}\times\frac{J_{n+1}(\omega)\omega^k}{n!\phi^{k+n}(\omega)}\frac{N^n}{\log^{k+n}N}(1+o(1)). \qquad (8)$$

（8）is called a unite prime formula in prime distribution. Let $n=1,k=0$, $J_2(\omega)=\phi(\omega)$. From (8) we have prime number theorem

$$\pi_1(N,2)=\left|\{P_1\le N:P_1 \text{ is prime}\}\right|=\frac{N}{\log N}(1+o(1)). \qquad (9)$$

Number theorists believe that there are infinitely many twin primes, but they do not have rigorous proof of this old conjecture by any method. All the prime theorems are conjectures except the prime number theorem, because they do not prove that prime equations have infinitely many prime solutions. We prove the following conjectures by this theorem.

Example 1. Twin primes $P, P+2$ (300BC).

From (6) and (7) we have Jiang's function

$$J_2(\omega)=\prod_{3\le P}(P-2)\ne 0.$$

Since $J_2(\omega)\ne 0$ in (2) exist infinitely many P prime equations such that $P+2$ is a prime equation. Therefore we prove that there are infinitely many primes P such that $P+2$ is a prime.

Let $\omega=30$ and $J_2(30)=3$. From (4) we have three P prime equations

$$P_3=30n+11,\quad P_5=30n+17,\quad P_8=30n+29.$$

From (8) we have the best asymptotic formula

$$\pi_2(N,2)=\left|\{P\le N:P+2 \text{ prime}\}\right|=\frac{J_2(\omega)\omega}{\phi^2(\omega)}\frac{N}{\log^2 N}(1+o(1))$$

$$=2\prod_{3\le P}\left(1-\frac{1}{(P-1)^2}\right)\frac{N}{\log^2 N}(1+o(1)).$$

In 1996 we proved twin primes conjecture [1]

Remark. $J_2(\omega)$ denotes the number of P prime equations, $\frac{\omega}{\phi^2(\omega)}\frac{N}{\log^2 N}(1+o(1))$ the number of solutions of primes for every P prime equation.

Example 2. Even Goldbach's conjecture $N=P_1+P_2$**.** Every even number $N\ge 6$ is the sum of two primes**.**

From (6) and (7) we have Jiang's function

$$J_2(\omega)=\prod_{3\le P}(P-2)\prod_{P|N}\frac{P-1}{P-2}\ne 0.$$

Since $J_2(\omega)\ne 0$ as $N\to\infty$ in (2) exist infinitely many P_1 prime equations such that $N-P_1$ is a prime equation. Therefore we prove that every even number $N\ge 6$ is the sum of two primes.

From (8) we have the best asymptotic formula

$$\pi_2(N,2)=\left|\{P_1\le N, N-P_1 \text{ prime}\}\right|=\frac{J_2(\omega)\omega}{\phi^2(\omega)}\frac{N}{\log^2 N}(1+o(1)).$$

$$=2\prod_{3\le P}\left(1-\frac{1}{(P-1)^2}\right)\prod_{P|N}\frac{P-1}{P-2}\frac{N}{\log^2 N}(1+o(1)).$$

In 1996 we proved even Goldbach's conjecture [1]

Example 3. Prime equations $P, P+2, P+6$.

From (6) and (7) we have Jiang's function

$$J_2(\omega)=\prod_{5\le P}(P-3)\neq 0,$$

$J_2(\omega)$ is denotes the number of P prime equations such that $P+2$ and $P+6$ are prime equations. Since $J_2(\omega)\neq 0$ in (2) exist infinitely many P prime equations such that $P+2$ and $P+6$ are prime equations. Therefore we prove that there are infinitely many primes P such that $P+2$ and $P+6$ are primes.

Let $\omega=30,\ J_2(30)=2$. From (4) we have two P prime equations

$$P_3=30n+11,\quad P_5=30n+17.$$

From (8) we have the best asymptotic formula

$$\pi_3(N,2)=\left|\{P\le N: P+2, P+6 \text{are primes}\}\right|=\frac{J_2(\omega)\omega^2}{\phi^3(\omega)}\frac{N}{\log^3 N}(1+o(1)).$$

Example 4. Odd Goldbach's conjecture $N=P_1+P_2+P_3$. Every odd number $N\ge 9$ is the sum of three primes.

From (6) and (7) we have Jiang's function

$$J_3(\omega)=\prod_{3\le P}\left(P^2-3P+3)\right)\prod_{P\mid N}\left(1-\frac{1}{P^2-3P+3}\right)\neq 0.$$

Since $J_3(\omega)\neq 0$ as $N\to\infty$ in (2) exist infinitely many pairs of P_1 and P_2 prime equations such that $N-P_1-P_2$ is a prime equation. Therefore we prove that every odd number $N\ge 9$ is the sum of three primes.

From (8) we have the best asymptotic formula

$$\pi_2(N,3)=\left|\{P_1,P_2\le N: N-P_1-P_2 \text{ prime}\}\right|=\frac{J_3(\omega)\omega}{2\phi^3(\omega)}\frac{N^2}{\log^3 N}(1+o(1))$$

$$=\prod_{3\le P}\left(1+\frac{1}{(P-1)^3}\right)\prod_{P\mid N}\left(1-\frac{1}{P^3-3P+3}\right)\frac{N^2}{\log^3 N}(1+o(1)).$$

Example 5. Prime equation $P_3=P_1P_2+2$.

From (6) and (7) we have Jiang's function

$$J_3(\omega)=\prod_{3\le P}\left(P^2-3P+2\right)\neq 0$$

$J_3(\omega)$ denotes the number of pairs of P_1 and P_2 prime equations such that P_3 is a prime equation. Since $J_3(\omega)\neq 0$ in (2) exist infinitely many pairs of P_1 and P_2 prime equations such that P_3 is a prime equation. Therefore we prove that there are infinitely many pairs of primes P_1 and P_2 such that P_3 is a prime.

From (8) we have the best asymptotic formula

$$\pi_2(N,3)=\left|\{P_1,P_2\le N: P_1P_2+2 \text{ prime}\}\right|=\frac{J_3(\omega)\omega}{4\phi^3(\omega)}\frac{N^2}{\log^3 N}(1+o(1)).$$

Note. deg $(P_1P_2)=2$.

Example 6 [12]. Prime equation $P_3=P_1^3+2P_2^3$.

From (6) and (7) we have Jiang's function

$$J_3(\omega)=\prod_{3\le P}\left[(P-1)^2-\chi(P)\right]\neq 0,$$

where $\chi(P)=3(P-1)$ if $2^{\frac{P-1}{3}}\equiv 1(\bmod P)$; $\chi(P)=0$ if $2^{\frac{P-1}{3}}\not\equiv 1(\bmod P)$; $\chi(P)=P-1$ otherwise.

Since $J_3(\omega)\neq 0$ in (2) there are infinitely many pairs of P_1 and P_2 prime equations such that P_3 is a prime equation. Therefore we prove that there are infinitely many pairs of primes P_1 and P_2 such that P_3 is a prime.

From (8) we have the best asymptotic formula

$$\pi_2(N,3)=\left|\{P_1,P_2\leq N: P_1^3+2P_2^3 \text{ prime}\}\right|=\frac{J_3(\omega)\omega}{6\phi^3(\omega)}\frac{N^2}{\log^3 N}(1+o(1)).$$

Example 7 [13]. Prime equation $P_3=P_1^4+(P_2+1)^2$.

From (6) and (7) we have Jiang's function

$$J_3(\omega)=\prod_{3\leq P}\left[(P-1)^2-\chi(P)\right]\neq 0$$

where $\chi(P)=2(P-1)$ if $P\equiv 1(\bmod 4)$; $\chi(P)=2(P-3)$ if $P\equiv 1(\bmod 8)$; $\chi(P)=0$ otherwise.

Since $J_3(\omega)\neq 0$ in (2) there are infinitely many pairs of P_1 and P_2 prime equations such that P_3 is a prime equation. Therefore we prove that there are infinitely many pairs of primes P_1 and P_2 such that P_3 is a prime.

From (8) we have the best asymptotic formula

$$\pi_2(N,3)=\left|\{P_1,P_2\leq N: P_3 \text{ prime}\}\right|=\frac{J_3(\omega)\omega}{8\phi^3(\omega)}\frac{N^2}{\log^3 N}(1+o(1)).$$

Example 8 [14-20]. Arithmetic progressions consisting only of primes. We define the arithmetic progressions of length k.

$$P_1, P_2=P_1+d, P_3=P_1+2d,\cdots,P_k=P_1+(k-1)d,(P_1,d)=1. \qquad (10)$$

From (8) we have the best asymptotic formula

$$\pi_2(N,2)=\left|\{P_1\leq N: P_1,P_1+d,\cdots,P_1+(k-1)d \text{ are primes}\}\right|$$

$$=\frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)}\frac{N}{\log^k N}(1+o(1)).$$

If $J_2(\omega)=0$ then (10) has finite prime solutions. If $J_2(\omega)\neq 0$ then there are infinitely many primes P_1 such that $P_2,\cdots,P_k$ are primes.

To eliminate d from (10) we have

$$P_3=2P_2-P_1,\quad P_j=(j-1)P_2-(j-2)P_1, 3\leq j\leq k.$$

From (6) and (7) we have Jiang's function

$$J_3(\omega)=\prod_{3\leq P<k}(P-1)\prod_{k\leq P}(P-1)(P-k+1)\neq 0$$

Since $J_3(\omega)\neq 0$ in (2) there are infinitely many pairs of P_1 and P_2 prime equations such that $P_3,\cdots,P_k$ are prime equations. Therefore we prove that there are infinitely many pairs of primes P_1 and P_2 such that $P_3,\cdots,P_k$ are primes.

From (8) we have the best asymptotic formula

$$\pi_{k-1}(N,3)=\left|\{P_1,P_2\leq N:(j-1)P_2-(j-2)P_1 \text{ prime}, 3\leq j\leq k\}\right|$$

$$=\frac{J_3(\omega)\omega^{k-2}}{2\phi^k(\omega)}\frac{N^2}{\log^k N}(1+o(1)) \qquad =\frac{1}{2}\prod_{2\le P<k}\frac{P^{k-2}}{(P-1)^{k-1}}\prod_{k\le P}\frac{P^{k-2}(P-k+1)}{(P-1)^{k-1}}\frac{N^2}{\log^k N}(1+o(1))$$
.

Example 9. It is a well-known conjecture that one of $P,P+2,P+2^2$ is always divisible by 3. To generalize above to the $k-$ primes, we prove the following conjectures. Let n be a square-free even number.

1. $P,P+n,P+n^2$,

where $3|(n+1)$.

From (6) and (7) we have $J_2(3)=0$, hence one of $P,P+n,P+n^2$ is always divisible by 3.

2. $P,P+n,P+n^2,\cdots,P+n^4$,

where $5|(n+b),b=2,3.$

From (6) and (7) we have $J_2(5)=0$, hence one of $P,P+n,P+n^2,\cdots,P+n^4$ is always divisible by 5.

3. $P,P+n,P+n^2,\cdots,P+n^6$,

where $7|(n+b),b=2,4.$

From (6) and (7) we have $J_2(7)=0$, hence one of $P,P+n,P+n^2,\cdots,P+n^6$ is always divisible by 7.

4. $P,P+n,P+n^2,\cdots,P+n^{10}$,

where $11|(n+b),b=3,4,5,9.$

From (6) and (7) we have $J_2(11)=0$, hence one of $P,P+n,P+n^2,\cdots,P+n^{10}$ is always divisible by 11.

5. $P,P+n,P+n^2,\cdots,P+n^{12}$,

where $13|(n+b),b=2,6,7,11.$

From (6) and (7) we have $J_2(13)=0$, hence one of $P,P+n,P+n^2,\cdots,P+n^{12}$ is always divisible by 13.

6. $P,P+n,P+n^2,\cdots,P+n^{16}$,

where $17|(n+b),b=3,5,6,7,10,11,12,14,15.$

From (6) and (7) we have $J_2(17)=0$, hence one of $P,P+n,P+n^2,\cdots,P+n^{16}$ is always divisible by 17.

7. $P,P+n,P+n^2,\cdots,P+n^{18}$,

where $19|(n+b),b=4,5,6,9,16.17.$

From (6) and (7) we have $J_2(19)=0$, hence one of $P,P+n,P+n^2,\cdots,P+n^{18}$ is always divisible by 19.

Example 10. Let n be an even number.

1. $P,P+n^i,i=1,3,5,\cdots,2k+1$,

From (6) and (7) we have $J_2(\omega)\ne 0$. Therefore we prove that there exist infinitely many primes P such that $P,P+n^i$ are primes for any k.

2. $P, P+n^i, i=2,4,6,\cdots,2k$.

From (6) and (7) we have $J_2(\omega)\neq 0$. Therefore we prove that there exist infinitely many primes P such that $P, P+n^i$ are primes for any k.

Example 11. Prime equation $2P_2 = P_1 + P_3$

From (6) and (7) we have Jiang's function

$$J_3(\omega)=\prod_{3\le P}(P^2-3P+2)\neq 0.$$

Since $J_3(\omega)\neq 0$ in (2) there are infinitely many pairs of P_1 and P_2 prime equations such that P_3 is prime equations. Therefore we prove that there are infinitely many pairs of primes P_1 and P_2 such that P_3 is a prime.

From (8) we have the best asymptotic formula

$$\pi_2(N,3)=\left|\{P_1,P_2\le N : P_3 \text{ prime}\}\right| = \frac{J_3(\omega)\omega}{2\phi^3(\omega)}\frac{N^2}{\log^3 N}(1+o(1)).$$

In the same way we can prove $2P_2^2 = P_3 + P_1$ which has the same Jiang's function.

Jiang's function is accurate sieve function. Using it we can prove any irreducible prime equations in prime distribution. There are infinitely many twin primes but we do not have rigorous proof of this old conjecture by any method [20]. As strong as the numerical evidence may be, we still do not even know whether there are infinitely many pairs of twin primes [21]. All the prime theorems are conjectures except the prime number theorem, because they do not prove the simplest twin primes. They conjecture that the prime distribution is randomness [12-25], because they do not understand theory of prime numbers.

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The Hardy-Littlewood prime *k*-tuple conjecture is false

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Abstract: Using Jiang function we prove Jiang prime k-tuple theorem. We prove that the Hardy-Littlewood prime k-tuple conjecture is false. Jiang prime k-tuple theorem can replace the Hardy-Littlewood prime k-tuple conjecture.

(A) Jiang prime k-tuple theorem [1, 2].

We define the prime k-tuple equation

$$p,\ p+n_i, \qquad (1)$$

where $2|n_i, i=1,\cdots k-1$.

we have Jiang function [1, 2]

$$J_2(\omega)=\prod_P(P-1-\chi(P)), \qquad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{i=1}^{k-1}(q+n_i)\equiv 0 \pmod P, \quad q=1,\cdots,p-1. \qquad (3)$$

If $\chi(P)<P-1$ then $J_2(\omega)\neq 0$. There exist infinitely many primes P such that each of $P+n_i$ is prime. If $\chi(P)=P-1$ then $J_2(\omega)=0$. There exist finitely many primes P such that each of $P+n_i$ is prime. $J_2(\omega)$ is a subset of Euler function $\phi(\omega)$ [2].

If $J_2(\omega)\neq 0$, then we hae the best asymptotic formula of the number of prime P [1, 2]

$$\pi_k(N,2)=\left|\{P\le N: P+n_i=prime\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)}\frac{N}{\log^k N}=C(k)\frac{N}{\log^k N} \qquad (4)$$

$$\phi(\omega)=\prod_{P}(P-1),$$

$$C(k)=\prod_{P}\left(1-\frac{1+\chi(P)}{P}\right)\left(1-\frac{1}{P}\right)^{-k}$$

（5）

Example 1. Let $k=2, P, P+2$, twin primes theorem.

From (3) we have

$$\chi(2)=0,\quad \chi(P)=1 \text{ if } P>2,$$

（6）

Substituting (6) into (2) we have

$$J_2(\omega)=\prod_{P\geq 3}(P-2)\neq 0$$

（7）

There exist infinitely many primes P such that $P+2$ is prime. Substituting (7) into (4) we have the best asymptotic pormula

$$\pi_k(N,2)=\left|\{P\leq N: P+2=prime\}\right|\sim 2\prod_{P\geq 3}\left(1-\frac{1}{(P-1)^2}\right)\frac{N}{\log^2 N}.$$

（8）

Example 2. Let $k=3, P, P+2, P+4$.

From (3) we have

$$\chi(2)=0,\quad \chi(3)=2$$

（9）

From (2) we have

$$J_2(\omega)=0.$$

（10）

It has only a solution $P=3$, $P+2=5$, $P+4=7$. One of $P, P+2, P+4$ is always divisible by 3.

Example 3. Let $k=4, P, P+n$, where $n=2,6,8$.

From (3) we have

$$\chi(2)=0, \chi(3)=1, \chi(P)=3 \text{ if } P>3.$$

（11）

Substituting (11) into (2) we have

$$J_2(\omega)=\prod_{P\geq 5}(P-4)\neq 0,$$

（12）

There exist infinitely many primes P such that each of $P+n$ is prime.

Substituting (12) into (4) we have the best asymptotic formula

$$\pi_4(N,2)=\left|\{P\leq N: P+n=prime\}\right|\sim \frac{27}{3}\prod_{P\geq 5}\frac{P^3(P-4)}{(P-1)^4}\frac{N}{\log^4 N}$$

（13）

Example 4. Let $k=5$, P, $P+n$, where $n=2,6,8,12$.

From (3) we have

$$\chi(2)=0, \chi(3)=1, \chi(5)=3, \chi(P)=4 \text{ if } P>5$$

（14）

Substituting (14) into (2) we have

$$J_2(\omega)=\prod_{P\geq 7}(P-5)\neq 0$$

（15）

There exist infinitely many primes P such that each of $P+n$ is prime. Substituting (15) into (4) we have the best asymptotic formula

$$\pi_5(N,2)=\left|\{P\leq N: P+n=prime\}\right|\sim \frac{15^4}{2^{11}}\prod_{P\geq 7}\frac{(P-5)P^4}{(P-1)^5}\frac{N}{\log^5 N}$$

（16）

Example 5. Let $k=6$, P, $P+n$, where $n=2,6,8,12,14$.

From (3) and (2) we have

$$\chi(2)=0,\ \chi(3)=1,\ \chi(5)=4,\quad J_2(5)=0 \tag{17}$$

It has only a solution $P=5$, $P+2=7$, $P+6=11$, $P+8=13$, $P+12=17$, $P+14=19$. One of $P+n$ is always divisible by 5.

（B）The Hardy-Littlewood prime k-tuple conjecture[3-14].

This conjecture is generally believed to be true,but has not been proved(Odlyzko et al.1999).

We define the prime k-tuple equation

$$P, P+n_i \tag{18}$$

where $2|n_i, i=1,\cdots,k-1$.

In 1923 Hardy and Littlewood conjectured the asymptotic formula

$$\pi_k(N,2)=\left|\{P\le N: P+n_i=prime\}\right|\sim H(k)\frac{N}{\log^k N}, \tag{19}$$

where

$$H(k)=\prod_P\left(1-\frac{\nu(P)}{P}\right)\left(1-\frac{1}{P}\right)^{-k} \tag{20}$$

$\nu(P)$ is the number of solutions of congruence

$$\prod_{i=1}^{k-1}(q+n_i)\equiv 0 \pmod P,\quad q=1,\cdots,P. \tag{21}$$

From (21) we have $\nu(P)<P$ and $H(k)\ne 0$. For any prime k-tuple equation there exist infinitely many primes P such that each of $P+n_i$ is prime, which is false.

Conjectore 1. Let $k=2, P, P+2$, twin primes theorem

Frome (21) we have

$$\nu(P)=1 \tag{22}$$

Substituting (22) into (20) we have

$$H(2)=\prod_P\frac{P}{P-1} \tag{23}$$

Substituting (23) into (19) we have the asymptotic formula

$$\pi_2(N,2)=\left|\{P\le N: P+2=prime\}\right|\sim\prod_P\frac{P}{P-1}\frac{N}{\log^2 N} \tag{24}$$

which is false see example 1.

Conjecture 2. Let $k=3, P, P+2, P+4$.

From (21) we have

$$\nu(2)=1,\ \nu(P)=2 \text{ if } P>2 \tag{25}$$

Substituting (25) into (20) we have

$$H(3)=4\prod_{P\ge3}\frac{P^2(P-2)}{(P-1)^3} \tag{26}$$

Substituting (26) into (19) we have asymptotic formula

$$\pi_3(N,2)=\left|\{P\le N: P+2=prime, P+4=prim\}\right|\sim 4\prod_{P\ge3}\frac{P^2(P-2)}{(P-1)^3}\frac{N}{\log^3 N} \tag{27}$$

which is false see example 2.

Conjecutre 3. Let $k=4$, $P, P+n$, where $n=2,6,8$.

From (21) we have

$$\nu(2)=1,\ \nu(3)=2,\ \nu(P)=3 \text{ if } P>3 \tag{28}$$

Substituting (28) into (20) we have

$$H(4)=\frac{27}{2}\prod_{P>3}\frac{P^3(P-3)}{(P-1)^4} \tag{29}$$

Substituting (29) into (19) we have asymptotic formula

$$\pi_4(N,2)=\left|\{P\le N: P+n=prime\}\right|\sim\frac{27}{2}\prod_{P>3}\frac{P^3(P-3)}{(P-1)^4}\frac{N}{\log^4 N} \tag{30}$$

Which is false see example 3.

Conjecture 4. Let $k=5,\ P, P+n$, where $n=2,6,8,12$

From (21) we have

$$\nu(2)=1,\ \nu(3)=2,\ \nu(5)=3,\ \nu(P)=4 \text{ if } P>5 \tag{31}$$

Substituting (31) into (20) we have

$$H(5)=\frac{15^4}{4^5}\prod_{P>5}\frac{P^4(P-4)}{(P-1)^5} \tag{32}$$

Substituting (32) into (19) we have asymptotic formula

$$\pi_5(N,2)=\left|\{P\le N: P+n=prime\}\right|\sim\frac{15^4}{4^5}\prod_{P>5}\frac{P^4(P-4)}{(P-1)^5}\frac{N}{\log^5 N} \tag{33}$$

Which is false see example 4.

Conjecutre 5. Let $k=6$, P, $P+n$, where $n=2,6,8,12,14$.

From (21) we have

$$\nu(2)=1,\ \nu(3)=2,\ \nu(5)=4,\ \nu(P)=5 \text{ if } P>5 \tag{34}$$

Substituting (34) into (20) we have

$$H(6)=\frac{15^5}{2^{13}}\prod_{P>5}\frac{(P-5)P^5}{(P-1)^6} \tag{35}$$

Substituting (35) into (19) we have asymptotic formula

$$\pi_6(N,2)=\left|\{P\le N: P+n=prime\}\right|\sim\frac{15^5}{2^{13}}\prod_{P>5}\frac{(P-5)P^5}{(P-1)^6}\frac{N}{\log^6 N} \tag{36}$$

which is false see example 5.

Conclusion. The Hardy-Littlewood prime k-tuple conjecture is false. The tool of addive prime number theory is basically the Hardy-Littlewood prime tuples conjecture. Jiang prime k-tuple theorem can replace Hardy-Littlewood prime k-tuple Conjecture. There cannot be really modern prime theory without Jiang function.

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Automorphic Functions And Fermat's Last Theorem(1)

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Abstract: In 1637 Fermat wrote: "*It is impossible to separate a cube into two cubes, or a biquadrate into two biquadrates, or in general any power higher than the second into powers of like degree: I have discovered a truly marvelous proof, which this margin is too small to contain.*"

This means: $x^n + y^n = z^n (n > 2)$ has no integer solutions, all different from 0(i.e., it has only the trivial solution, where one of the integers is equal to 0). It has been called Fermat's last theorem (FLT). It suffices to prove FLT for exponent 4. and every prime exponent P. Fermat proved FLT for exponent 4. Euler proved FLT for exponent 3.

In this paper using automorphic functions we prove FLT for exponents $3P$ and P, where P is an odd prime. The proof of FLT must be direct. But indirect proof of FLT is disbelieving.

In 1974 Jiang found out Euler formula of the cyclotomic real numbers in the cyclotomic fields

$$\exp\left(\sum_{i=1}^{n-1} t_i J^i\right) = \sum_{i=1}^{n} S_i J^{i-1} \tag{1}$$

where J denotes a n th root of unity, $J^n = 1$, n is an odd number, t_i are the real numbers. S_i is called the automorphic functions(complex hyperbolic functions) of order n with $n-1$ variables [1-7].

$$S_i = \frac{1}{n}[e^A + 2\sum_{j=1}^{\frac{n-1}{2}} (-1)^{(i-1)j} e^{B_j} \cos(\theta_j + (-1)^j \frac{(i-1)j\pi}{n})] \tag{2}$$

where i=1,2,…,n;

$$A = \sum_{\alpha-1}^{n-1} t_\alpha, \qquad B_j = \sum_{\alpha=1}^{n-1} t_\alpha (-1)^{\alpha j} \cos\frac{\alpha j\pi}{n}, \tag{3}$$

$$\theta_j=(-1)^{j+1}\sum_{\alpha=1}^{n-1} t_\alpha(-1)^{\alpha j}\sin\frac{\alpha j\pi}{n}, \qquad A+2\sum_{j=1}^{\frac{n-1}{2}} B_j=0$$

(2) may be written in the matrix form

$$\begin{bmatrix} S_1\\ S_2\\ S_3\\ \cdots\\ S_n \end{bmatrix}=\frac{1}{n}\begin{bmatrix} 1 & 1 & 0 & \cdots & 0\\ 1 & -\cos\frac{\pi}{n} & -\sin\frac{\pi}{n} & \cdots & -\sin\frac{(n-1)\pi}{2n}\\ 1 & \cos\frac{2\pi}{n} & \sin\frac{2\pi}{n} & \cdots & -\sin\frac{(n-1)\pi}{n}\\ \cdots & \cdots & \cdots & \cdots & \cdots\\ 1 & \cos\frac{(n-1)\pi}{n} & \sin\frac{(n-1)\pi}{n} & \cdots & -\sin\frac{(n-1)^2\pi}{2n} \end{bmatrix}\begin{bmatrix} e^A\\ 2e^{B_1}\cos\theta_1\\ 2e^{B_1}\sin\theta_1\\ \cdots\\ 2\exp B_{\frac{n-1}{2}}\sin\theta_{\frac{n-1}{2}} \end{bmatrix} \quad (4)$$

where $(n-1)/2$ is an even number.

From (4) we have its inverse transformation

$$\begin{bmatrix} e^A\\ e^{B_1}\cos\theta_1\\ e^{B_1}\sin\theta_1\\ \cdots\\ \exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix}=\begin{bmatrix} 1 & 1 & 1 & \cdots & 1\\ 1 & -\cos\frac{\pi}{n} & \cos\frac{2\pi}{n} & \cdots & \cos\frac{(n-1)\pi}{n}\\ 0 & -\sin\frac{\pi}{n} & \sin\frac{2\pi}{n} & \cdots & \sin\frac{(n-1)\pi}{n}\\ \cdots & \cdots & \cdots & \cdots & \cdots\\ 0 & -\sin\frac{(n-1)\pi}{2n} & -\sin\frac{(n-1)\pi}{n} & \cdots & -\sin\frac{(n-1)^2\pi}{2n} \end{bmatrix}\begin{bmatrix} S_1\\ S_2\\ S_3\\ \cdots\\ S_n \end{bmatrix} \quad (5)$$

From (5) we have

$$e^A=\sum_{i=1}^{n} S_i, \qquad e^{B_j}\cos\theta_j=S_1+\sum_{i=1}^{n-1} S_{1+i}(-1)^{ij}\cos\frac{ij\pi}{n}$$

$$e^{B_j}\sin\theta_j=(-1)^{j+1}\sum_{i=1}^{n-1} S_{1+i}(-1)^{ij}\sin\frac{ij\pi}{n}, \qquad (6)$$

In (3) and (6) t_i and S_i have the same formulas. (4) and (5) are the most critical formulas of proofs for FLT. Using (4) and (5) in 1991 Jiang invented that every factor of exponent n has the Fermat equation and proved FLT [1-7] Substituting (4) into (5) we prove (5).

$$\begin{bmatrix} e^A\\ e^{B_1}\cos\theta_1\\ e^{B_1}\sin\theta_1\\ \cdots\\ \exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix}=\frac{1}{n}\begin{bmatrix} 1 & 1 & 1 & \cdots & 1\\ 1 & -\cos\frac{\pi}{n} & \cos\frac{2\pi}{n} & \cdots & \cos\frac{(n-1)\pi}{n}\\ 0 & -\sin\frac{\pi}{n} & \sin\frac{2\pi}{n} & \cdots & \sin\frac{(n-1)\pi}{n}\\ \cdots & \cdots & \cdots & \cdots & \cdots\\ 0 & -\sin\frac{(n-1)\pi}{2n} & -\sin\frac{(n-1)\pi}{n} & \cdots & -\sin\frac{(n-1)^2\pi}{2n} \end{bmatrix}\times$$

$$\begin{bmatrix} 1 & 1 & 0 & \cdots & 0 \\ 1 & -\cos\dfrac{\pi}{n} & -\sin\dfrac{\pi}{n} & \cdots & -\sin\dfrac{(n-1)\pi}{2n} \\ 1 & \cos\dfrac{2\pi}{n} & \sin\dfrac{2\pi}{n} & \cdots & -\sin\dfrac{(n-1)\pi}{n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 1 & \cos\dfrac{(n-1)\pi}{n} & \sin\dfrac{(n-1)\pi}{n} & \cdots & -\sin\dfrac{(n-1)^2\pi}{2n} \end{bmatrix} \begin{bmatrix} e^A \\ 2e^{B_1}\cos\theta_1 \\ 2e^{B_1}\sin\theta_1 \\ \cdots \\ 2\exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix}$$

$$= \frac{1}{n}\begin{bmatrix} n & 0 & 0 & \cdots & 0 \\ 0 & \dfrac{n}{2} & 0 & \cdots & 0 \\ 0 & 0 & \dfrac{n}{2} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & \dfrac{n}{2} \end{bmatrix} \begin{bmatrix} e^A \\ 2e^{B_1}\cos\theta_1 \\ 2e^{B_1}\sin\theta_1 \\ \cdots \\ 2\exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix}$$

$$= \begin{bmatrix} e^A \\ e^{B_1}\cos\theta_1 \\ e^{B_1}\sin\theta_1 \\ \cdots \\ \exp(B_{\frac{n-1}{2}})\sin(\theta_{\frac{n-1}{2}}) \end{bmatrix}, \tag{7}$$

where $1+\sum\limits_{j=1}^{n-1}(\cos\frac{j\pi}{n})^2=\frac{n}{2}$, $\sum\limits_{j=1}^{n-1}(\sin\frac{j\pi}{n})^2=\frac{n}{2}$.

From (3) we have

$$\exp(A+2\sum_{j=1}^{\frac{n-1}{2}}B_j)=1 \, . \tag{8}$$

From (6) we have

$$\exp(A+2\sum_{j=1}^{\frac{n-1}{2}}B_j)=\begin{vmatrix} S_1 & S_n & \cdots & S_2 \\ S_2 & S_1 & \cdots & S_3 \\ \cdots & \cdots & \cdots & \cdots \\ S_n & S_{n-1} & \cdots & S_1 \end{vmatrix} = \begin{vmatrix} S_1 & (S_1)_1 & \cdots & (S_1)_{n-1} \\ S_2 & (S_2)_1 & \cdots & (S_2)_{n-1} \\ \cdots & \cdots & \cdots & \cdots \\ S_n & (S_n)_1 & \cdots & (S_n)_{n-1} \end{vmatrix}, \tag{9}$$

where $(S_i)_j=\frac{\partial S_i}{\partial t_j}$ [7].

From (8) and (9) we have the circulant determinant

$$\exp(A+2\sum_{j=1}^{\frac{n-1}{2}}B_j)=\begin{vmatrix} S_1 & S_n & \cdots & S_2 \\ S_2 & S_1 & \cdots & S_3 \\ \cdots & \cdots & \cdots & \vdots \\ S_n & S_{n-1} & \cdots & S_1 \end{vmatrix}=1 \qquad （10）$$

If $S_i \neq 0$, where $i=1,2,\cdots,n$, then (10) has infinitely many rational solutions.

Assume $S_1 \neq 0$, $S_2 \neq 0$, $S_i = 0$ where $i=3,4,\cdots,n.\ S_i=0$ are $n-2$ indeterminate equations with $n-1$ variables. From (6) we have

$$e^A=S_1+S_2,\quad e^{2B_j}=S_1^2+S_2^2+2S_1S_2(-1)^j\cos\frac{j\pi}{n}. \qquad （11）$$

From (10) and (11) we have the Fermat equation

$$\exp(A+2\sum_{j=1}^{\frac{n-1}{2}}B_j)=(S_1+S_2)\prod_{j=1}^{\frac{n-1}{2}}(S_1^2+S_2^2+2S_1S_2(-1)^j\cos\frac{j\pi}{n})=S_1^n+S_2^n=1 \qquad （12）$$

Example[1]. Let $n=15$. From (3) we have

$$A=(t_1+t_{14})+(t_2+t_{13})+(t_3+t_{12})+(t_4+t_{11})+(t_5+t_{10})+(t_6+t_9)+(t_7+t_8)$$

$$B_1=-(t_1+t_{14})\cos\frac{\pi}{15}+(t_2+t_{13})\cos\frac{2\pi}{15}-(t_3+t_{12})\cos\frac{3\pi}{15}+(t_4+t_{11})\cos\frac{4\pi}{15}$$
$$-(t_5+t_{10})\cos\frac{5\pi}{15}+(t_6+t_9)\cos\frac{6\pi}{15}-(t_7+t_8)\cos\frac{7\pi}{15},$$

$$B_2=(t_1+t_{14})\cos\frac{2\pi}{15}+(t_2+t_{13})\cos\frac{4\pi}{15}+(t_3+t_{12})\cos\frac{6\pi}{15}+(t_4+t_{11})\cos\frac{8\pi}{15}$$
$$+(t_5+t_{10})\cos\frac{10\pi}{15}+(t_6+t_9)\cos\frac{12\pi}{15}+(t_7+t_8)\cos\frac{14\pi}{15},$$

$$B_3=-(t_1+t_{14})\cos\frac{3\pi}{15}+(t_2+t_{13})\cos\frac{6\pi}{15}-(t_3+t_{12})\cos\frac{9\pi}{15}+(t_4+t_{11})\cos\frac{12\pi}{15}$$
$$-(t_5+t_{10})\cos\frac{15\pi}{15}+(t_6+t_9)\cos\frac{18\pi}{15}-(t_7+t_8)\cos\frac{21\pi}{15},$$

$$B_4=(t_1+t_{14})\cos\frac{4\pi}{15}+(t_2+t_{13})\cos\frac{8\pi}{15}+(t_3+t_{12})\cos\frac{12\pi}{15}+(t_4+t_{11})\cos\frac{16\pi}{15}$$
$$+(t_5+t_{10})\cos\frac{20\pi}{15}+(t_6+t_9)\cos\frac{24\pi}{15}+(t_7+t_8)\cos\frac{28\pi}{15},$$

$$B_5=-(t_1+t_{14})\cos\frac{5\pi}{15}+(t_2+t_{13})\cos\frac{10\pi}{15}-(t_3+t_{12})\cos\frac{15\pi}{15}+(t_4+t_{11})\cos\frac{20\pi}{15}$$
$$-(t_5+t_{10})\cos\frac{25\pi}{15}+(t_6+t_9)\cos\frac{30\pi}{15}-(t_7+t_8)\cos\frac{35\pi}{15},$$

$$B_6=(t_1+t_{14})\cos\frac{6\pi}{15}+(t_2+t_{13})\cos\frac{12\pi}{15}+(t_3+t_{12})\cos\frac{18\pi}{15}+(t_4+t_{11})\cos\frac{24\pi}{15}$$
$$+(t_5+t_{10})\cos\frac{30\pi}{15}+(t_6+t_9)\cos\frac{36\pi}{15}+(t_7+t_8)\cos\frac{42\pi}{15},$$

$$B_7=-(t_1+t_{14})\cos\frac{7\pi}{15}+(t_2+t_{13})\cos\frac{14\pi}{15}-(t_3+t_{12})\cos\frac{21\pi}{15}+(t_4+t_{11})\cos\frac{28\pi}{15}$$
$$-(t_5+t_{10})\cos\frac{35\pi}{15}+(t_6+t_9)\cos\frac{42\pi}{15}-(t_7+t_8)\cos\frac{49\pi}{15},$$

$$A+2\sum_{j=1}^{7}B_j=0, \qquad A+2B_3+2B_6=5(t_5+t_{10}) . \tag{13}$$

Form (12) we have the Fermat equation

$$\exp(A+2\sum_{j=1}^{7}B_j)=S_1^{15}+S_2^{15}=(S_1^5)^3+(S_2^5)^3=1 . \tag{14}$$

From (13) we have

$$\exp(A+2B_3+2B_6)=[\exp(t_5+t_{10})]^5 . \tag{15}$$

From (11) we have

$$\exp(A+2B_3+2B_6)=S_1^5+S_2^5 . \tag{16}$$

From (15) and (16) we have the Fermat equation

$$\exp(A+2B_3+2B_6)=S_1^5+S_2^5=[\exp(t_5+t_{10})]^5 . \tag{17}$$

Euler proved that (14) has no rational solutions for exponent 3[8]. Therefore we prove that (17) has no rational solutions for exponent 5[1].

Theorem 1. [1-7]. Let $n=3P$,where $P>3$ is odd prime. From (12) we have the Fermat's equation

$$\exp(A+2\sum_{j=1}^{3P-1}B_j)=S_1^{3P}+S_2^{3P}=(S_1^P)^3+(S_2^P)^3=1 . \tag{18}$$

From (3) we have

$$\exp(A+2\sum_{j=1}^{\frac{P-1}{2}}B_{3j})=[\exp(t_P+t_{2P})]^P . \tag{19}$$

From (11) we have

$$\exp(A+2\sum_{j=1}^{\frac{P-1}{2}}B_{3j})=S_1^P+S_2^P . \tag{20}$$

From (19) and (20) we have the Fermat equation

$$\exp(A+2\sum_{j=1}^{\frac{P-1}{2}}B_{3j})=S_1^P+S_2^P=[\exp(t_P+t_{2P})]^P . \tag{21}$$

Euler proved that (18) has no rational solutions for exponent 3[8]. Therefore we prove that (21) has no rational solutions for $P>3$ [1, 3-7].

Theorem 2. In 1847 Kummer write the Fermat's equation

$$x^P+y^P=z^P \tag{22}$$

in the form

$$(x+y)(x+ry)(x+r^2y)\cdots(x+r^{P-1}y)=z^P \tag{23}$$

where P is odd prime, $r=\cos\frac{2\pi}{P}+i\sin\frac{2\pi}{P}$.

Kummer assume the divisor of each factor is a P th power. Kummer proved FLT for prime exponent p<100

[8]..

We consider the Fermat's equation

$$x^{3P}+y^{3P}=z^{3P} \quad （24）$$

we rewrite (24)

$$(x^P)^3+(y^P)^3=(z^P)^3 \quad (25)$$

From (24) we have

$$(x^P+y^P)(x^P+ry^P)(x^P+r^2y^P)=z^{3P} \quad （26）$$

where $r=\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}$

We assume the divisor of each factor is a P th power.

Let $S_1=\frac{x}{z}$, $S_2=\frac{y}{z}$. From (20) and (26) we have the Fermat's equation

$$x^P+y^P=[z\times\exp(t_P+t_{2P})]^P \quad (27)$$

Euler proved that (25) has no integer solutions for exponent 3[8]. Therefore we prove that (27) has no integer solutions for prime exponent P.

Fermat Theorem. It suffices to prove FLT for exponent 4. We rewrite (24)

$$(x^3)^P+(y^3)^P=(z^3)^P \quad (28)$$

Euler proved that（25）has no integer solutions for exponent 3 [8]. Therefore we prove that (28) has no integer solutions for all prime exponent P [1-7].

We consider Fermat equation

$$x^{4P}+y^{4P}=z^{4P} \quad (29)$$

We rewrite (29)

$$(x^P)^4+((y^P)^4=(z^P)^4 \quad （30）$$

$$(x^4)^P+(y^4)^P=(z^4)^P \quad （31）$$

Fermat proved that (30) has no integer solutions for exponent 4 [8]. Therefore we prove that (31) has no integer solutions for all prime exponent P [2,5,7].This is the proof that Fermat thought to have had.

Remark. It suffices to prove FLT for exponent 4. Let $n=4P$, where P is an odd prime. We have the Fermat's equation for exponent $4P$ and the Fermat's equation for exponent P [2,5,7]. This is the proof that Fermat thought to have had. In complex hyperbolic functions let exponent n be $n=\Pi P$, $n=2\Pi P$ and $n=4\Pi P$. Every factor of exponent n has the Fermat's equation [1-7]. In complex trigonometric functions let exponent n be $n=\Pi P$, $n=2\Pi P$ and $n=4\Pi P$. Every factor of exponent n has Fermat's equation [1-7].Using modular elliptic curves Wiles and Taylor prove FLT[9,10].This is not the proof that Fermat thought to have had. The classical theory of automorphic functions, created by Klein and Poincare, was concerned with the study of analytic functions in the unit circle that are invariant under a discrete group of transformations. Automorphic functions are generalization of the trigonometric,hyperbolic,elliptic, and certain other functions of elementary analysis. The complex trigonometric functions and complex hyperbolic functions have a wide application in mathematics and physics.

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Automorphic Functions And Fermat's Last Theorem（2）

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Abstract: In 1637 Fermat wrote: "*It is impossible to separate a cube into two cubes, or a biquadrate into two biquadrates, or in general any power higher than the second into powers of like degree: I have discovered a truly marvelous proof, which this margin is too small to contain.*"

This means: $x^n + y^n = z^n (n > 2)$ has no integer solutions, all different from 0(i.e., it has only the trivial solution, where one of the integers is equal to 0). It has been called Fermat's last theorem (FLT). It suffices to prove FLT for exponent 4. and every prime exponent P. Fermat proved FLT for exponent 4. Euler proved FLT for exponent 3.

In this paper using automorphic functions we prove FLT for exponents $6P$ and P, where P is an odd prime. The proof of FLT must be direct .But indirect proof of FLT is disbelieving.

In 1974 Jiang found out Euler formula of the cyclotomic real numbers in the cyclotomic fields

$$\exp\left(\sum_{i=1}^{2n-1} t_i J^i\right) = \sum_{i=1}^{2n} S_i J^{i-1} \qquad (1)$$

where J denotes a $2n$ th root of unity, $J^{2n} = 1$, n is an odd number, t_i are the real numbers.

S_i is called the automorphic functions(complex hyperbolic functions) of order $2n$ with $2n-1$ variables [5,7].

$$S_i = \frac{1}{2n}\left[e^{A_1} + 2\sum_{j=1}^{\frac{n-1}{2}} (-1)^{(i-1)jB_j} \cos\left(\theta_j + (-1)^j \frac{(i-1)j\pi}{n}\right)\right]$$

$$+\frac{(-1)^{(i-1)}}{2n}\left[e^{A_2} + 2\sum_{j=1}^{\frac{n-1}{2}} (-1)^{(i-1)j} e^{D_j} \cos\left(\phi_j + (-1)^{j+1} \frac{(i-1)j\pi}{n}\right)\right], \qquad (2)$$

where $i = 1, \ldots, 2n$;

$$A_1=\sum_{\alpha=1}^{2n-1} t_\alpha,\quad B_j=\sum_{\alpha=1}^{2n-1} t_\alpha(-1)^{\alpha j}\cos\frac{\alpha j\pi}{n},\theta_j=(-1)^{(j+1)}\sum_{\alpha=1}^{2n-1} t_\alpha(-1)^{\alpha j}\sin\frac{\alpha j\pi}{n},$$

$$A_2=\sum_{\alpha=1}^{2n-1} t_\alpha(-1)^{\alpha},\quad D_j=\sum_{\alpha=1}^{2n-1} t_\alpha(-1)^{(j-1)\alpha}\cos\frac{\alpha j\pi}{n},$$

$$\phi_j=(-1)^{j}\sum_{\alpha=1}^{2n-1} t_\alpha(-1)^{(j-1)\alpha}\sin\frac{\alpha j\pi}{n},A_1+A_2+2\sum_{j=1}^{\frac{n-1}{2}}(B_j+D_j)=0$$

（3）

From (2) we have its inverse transformation[5,7]

$$e^{A_1}=\sum_{i=1}^{2n} S_i,\quad e^{A_2}=\sum_{i=1}^{2n} S_i(-1)^{1+i}$$

$$e^{B_j}\cos\theta_j=S_1+\sum_{i=1}^{2n-1} S_{1+i}(-1)^{ij}\cos\frac{ij\pi}{n},$$

$$e^{B_j}\sin\theta_j=(-1)^{(j+1)}\sum_{i=1}^{2n-1} S_{1+i}(-1)^{ij}\sin\frac{ij\pi}{n},$$

$$e^{D_j}\cos\phi_j=S_1+\sum_{i=1}^{2n-1} S_{1+i}(-1)^{(j-1)i}\cos\frac{ij\pi}{n}$$

$$e^{D_j}\sin\phi_j=(-1)^{j}\sum_{i=1}^{2n-1} S_{1+i}(-1)^{(j-1)i}\sin\frac{ij\pi}{n}$$

（4）

(3) and (4) have the same form.
From (3) we have

$$\exp\left[A_1+A_2+2\sum_{j=1}^{\frac{n-1}{2}}(B_j+D_j)\right]=1$$

（5）

From (4) we have

$$\exp\left[A_1+A_2+2\sum_{j=1}^{\frac{n-1}{2}}(B_j+D_j)\right]=\begin{vmatrix} S_1 & S_{2n} & \cdots & S_2 \\ S_2 & S_1 & \cdots & S_3 \\ \cdots & \cdots & \cdots & \cdots \\ S_{2n} & S_{2n-1} & \cdots & S_1 \end{vmatrix}$$

$$=\begin{vmatrix} S_1 & (S_1)_1 & \cdots & (S_1)_{2n-1} \\ S_2 & (S_2)_1 & \cdots & (S_2)_{2n-1} \\ \cdots & \cdots & \cdots & \cdots \\ S_{2n} & (S_{2n})_1 & \cdots & (S_{2n})_{2n-1} \end{vmatrix}$$

(6)

where $(S_i)_j=\frac{\partial S_i}{\partial t_j}$ [7]..
From (5) and (6) we have circulant determinant

$$\exp\left[A_1 + A_2 + 2\sum_{j=1}^{\frac{n-1}{2}} (B_j + D_j)\right] = \begin{vmatrix} S_1 & S_{2n} & \cdots & S_2 \\ S_2 & S_1 & \cdots & S_3 \\ \cdots & \cdots & \cdots & \cdots \\ S_{2n} & S_{2n-1} & \cdots & S_1 \end{vmatrix} = 1 \tag{7}$$

If $S_i \neq 0$, where $i = 1,2,3,\ldots,2n$, then (7) have infinitely many rational solutions.

Let $n = 1$. From (3) we have $A_1 = t_1$ and $A_2 = -t_1$. From (2) we have

$$S_1 = \operatorname{ch} t_1 \qquad S_2 = \operatorname{sh} t_1 \tag{8}$$

we have Pythagorean theorem

$$\operatorname{ch}^2 t_1 - \operatorname{sh}^2 t_1 = 1 \tag{9}$$

(9) has infinitely many rational solutions.

Assume $S_1 \neq 0, S_2 \neq 0, S_i \neq 0$, where $i = 3,\ldots,2n$. $S_i = 0$ are $(2n-2)$ indeterminate equations with $(2n-1)$ variables. From (4) we have

$$e^{A_1} = S_1 + S_2, \quad e^{A_2} = S_1 - S_2, e^{2B_j} = S_1^2 + S_2^2 + 2S_1S_2(-1)^j \cos\frac{j\pi}{n},$$

$$e^{2D_j} = S_1^2 + S_2^2 + 2S_1S_2(-1)^{j+1} \cos\frac{j\pi}{n} \tag{10}$$

Example. Let $n = 15$. From (3) and (10) we have Fermat's equation

$$\exp[A_1 + A_2 + 2\sum_{j=1}^{7} (B_j + D_j)] = S_1^{30} - S_2^{30} = (S_1^{10})^3 - (S_2^{10})^3 = 1 \tag{11}$$

From (3) we have

$$\exp(A_1 + 2B_3 + 2B_6) = [\exp(\sum_{j=1}^{5} t_{5j})]^5 \tag{12}$$

From (10) we have

$$\exp(A_1 + 2B_3 + 2B_6) = S_1^5 + S_2^5 \tag{13}$$

From (12) and (13) we have Fermat's equation

$$\exp(A_1 + 2B_3 + 2B_6) = S_1^5 + S_2^5 = [\exp(\sum_{j=1}^{5} t_{5j})]^5 \tag{14}$$

Euler prove that (19) has no rational solutions for exponent 3 [8]. Therefore we prove that (14) has no rational solutions for exponent 5.

Theorem. Let $n = 3P$ where P is an odd prime. From (7) and (8) we have Fermat's equation

$$\exp(A_1 + A_2 + 2\sum_{j=1}^{\frac{3P-1}{2}} (B_j + D_j)] = S_1^{6P} - S_2^{6P} = (S_1^{2P})^3 - (S_2^{2P})^3 = 1 \tag{15}$$

From (3) we have

$$\exp\left(A_1 + 2\sum_{j=1}^{\frac{P-1}{2}} B_{3j}\right) = \left[\exp\left(\sum_{j=1}^{5} t_{jP}\right)\right]^P \tag{16}$$

From (10) we have

$$\exp\left(A_1 + 2\sum_{j=1}^{\frac{P-1}{2}} B_{3j}\right) = S_1^P + S_2^P \tag{17}$$

From (16) and (17) we have Fermat's equation

$$\exp\left(A_1 + 2\sum_{j=1}^{\frac{P-1}{2}} B_{3j}\right) = S_1^P + S_2^P = \left[\exp\left(\sum_{j=1}^{5} t_{jP}\right)\right]^P \tag{18}$$

Euler prove that (15) has no rational solutions for exponent 3[8]. Therefore we prove that (18) has no rational solutions for prime exponent P [5,7].

Remark. It suffices to prove FLT for exponent 4. Let $n = 4P$, where P is an odd prime. We have the Fermat's equation for exponent $4P$ and the Fermat's equation for exponent P [2,5,7]. This is the proof that Fermat thought to have had. In complex hyperbolic functions let exponent n be $n = \Pi P$, $n = 2\Pi P$ and $n = 4\Pi P$. Every factor of exponent n has the Fermat's equation [1-7]. In complex trigonometric functions let exponent n be $n = \Pi P$, $n = 2\Pi P$ and $n = 4\Pi P$. Every factor of exponent n has Fermat's equation [1-7]. Using modular elliptic curves Wiles and Taylor prove FLT [9, 10]. This is not the proof that Fermat thought to have had. The classical theory of automorphic functions, created by Klein and Poincare, was concerned with the study of analytic functions in the unit circle that are invariant under a discrete group of transformation. Automorphic functions are the generalization of trigonometric, hyperbolic, elliptic, and certain other functions of elementary analysis. The complex trigonometric functions and complex hyperbolic functions have a wide application in mathematics and physics.

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Automorphic Functions And Fermat's Last Theorem（3）(Fermat's Proof of FLT)

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Abstract: In 1637 Fermat wrote: "*It is impossible to separate a cube into two cubes, or a biquadrate into two biquadrates, or in general any power higher than the second into powers of like degree: I have discovered a truly marvelous proof, which this margin is too small to contain.*"

This means: $x^n + y^n = z^n (n > 2)$ has no integer solutions, all different from 0(i.e., it has only the trivial solution, where one of the integers is equal to 0). It has been called Fermat's last theorem (FLT). It suffices to prove FLT for exponent 4 and every prime exponent P. Fermat proved FLT for exponent 4. Euler proved FLT for exponent 3.

In this paper using automorphic functions we prove FLT for exponents $4P$ and P, where P is an odd prime. We rediscover the Fermat proof. The proof of FLT must be direct. But indirect proof of FLT is disbelieving.

In 1974 Jiang found out Euler formula of the cyclotomic real numbers in the cyclotomic fields

$$\exp\left(\sum_{i=1}^{4m-1} t_i J^i\right) = \sum_{i=1}^{4m} S_i J^{i-1}, \tag{1}$$

where J denotes a $4m$ th root of unity, $J^{4m} = 1$, m=1,2,3,…, t_i are the real numbers. S_i is called the automorphic functions(complex hyperbolic functions) of order $4m$ with $4m-1$ variables [2,5,7].

$$S_i = \frac{1}{4m}\left[e^{A_1} + 2e^H \cos\left(\beta + \frac{(i-1)\pi}{2}\right) + 2\sum_{j=1}^{m-1} e^{B_j} \cos\left(\theta_j + \frac{(i-1)j\pi}{2m}\right)\right]$$
$$+ \frac{(-1)^{(i-1)}}{4m}\left[e^{A_2} + 2\sum_{j=1}^{m-1} e^{D_j} \cos\left(\phi_j - \frac{(i-1)j\pi}{2m}\right)\right] \tag{2}$$

where $i = 1, \ldots, 4m$;

$$A_1 = \sum_{\alpha=1}^{4m-1} t_\alpha, \quad A_2 = \sum_{\alpha=1}^{4m-1} t_\alpha (-1)^\alpha, \qquad H = \sum_{\alpha=1}^{2m-1} t_{2\alpha}(-1)^\alpha, \quad \beta = \sum_{\alpha=1}^{2m} t_{2\alpha-1}(-1)^\alpha,$$

$$B_j = \sum_{\alpha=1}^{4m-1} t_\alpha \cos\frac{\alpha j\pi}{2m}, \quad \theta_j = -\sum_{\alpha=1}^{4m-1} t_\alpha \sin\frac{\alpha j\pi}{2m},$$

$$D_j = \sum_{\alpha=1}^{4m-1} t_\alpha (-1)^\alpha \cos\frac{\alpha j\pi}{2m}, \quad \phi_j = \sum_{\alpha=1}^{4m-1} t_\alpha (-1)^\alpha \sin\frac{\alpha j\pi}{2m},$$

$$A_1 + A_2 + 2H + 2\sum_{j=1}^{m-1} (B_j + D_j) = 0. \tag{3}$$

From (2) we have its inverse transformation[5,7]

$$e^{A_1} = \sum_{i=1}^{4m} S_i, \quad e^{A_2} = \sum_{i=1}^{4m} S_i (-1)^{1+i}$$

$$e^{H}\cos\beta=\sum_{i=1}^{2m}S_{2i-1}(-1)^{1+i},\quad e^{H}\sin\beta=\sum_{i=1}^{2m}S_{2i}(-1)^{i},$$

$$e^{B_j}\cos\theta_j=S_1+\sum_{i=1}^{4m-1}S_{1+i}\cos\frac{ij\pi}{2m},\quad e^{B_j}\sin\theta_j=-\sum_{i=1}^{4m-1}S_{1+i}\sin\frac{ij\pi}{2m},$$

$$e^{D_j}\cos\phi_j=S_1+\sum_{i=1}^{4m-1}S_{1+i}(-1)^{i}\cos\frac{ij\pi}{2m},\quad e^{D_j}\sin\phi_j=\sum_{i=1}^{4m-1}S_{1+i}(-1)^{i}\sin\frac{ij\pi}{2m}. \tag{4}$$

(3) and (4) have the same form.

From (3) we have

$$\exp\left[A_1+A_2+2H+2\sum_{j=1}^{m-1}(B_j+D_j)\right]=1 \tag{5}$$

From (4) we have

$$\exp\left[A_1+A_2+2H+2\sum_{j=1}^{m-1}(B_j+D_j)\right]=\begin{vmatrix}S_1 & S_{4m} & \cdots & S_2\\ S_2 & S_1 & \cdots & S_3\\ \cdots & \cdots & \cdots & \cdots\\ S_{4m} & S_{4m-1} & \cdots & S_1\end{vmatrix}$$

$$=\begin{vmatrix}S_1 & (S_1)_1 & \cdots & (S_1)_{4m-1}\\ S_2 & (S_2)_1 & \cdots & (S_2)_{4m-1}\\ \cdots & \cdots & \cdots & \cdots\\ S_{4m} & (S_{4m})_1 & \cdots & (S_{4m})_{4m-1}\end{vmatrix} \tag{6}$$

where

$$(S_i)_j=\frac{\partial S_i}{\partial t_j}$$ [7]

From (5) and (6) we have circulant determinant

$$\exp\left[A_1+A_2+2H+2\sum_{j=1}^{m-1}(B_j+D_j)\right]=\begin{vmatrix}S_1 & S_{4m} & \cdots & S_2\\ S_2 & S_1 & \cdots & S_3\\ \cdots & \cdots & \cdots & \cdots\\ S_{4m} & S_{4m-1} & \cdots & S_1\end{vmatrix}=1 \tag{7}$$

Assume $S_1\neq 0, S_2\neq 0, S_i=0$, where $i=3,\ldots,4m.$ $S_i=0$ are $(4m-2)$ indeterminate equations with $(4m-1)$ variables. From (4) we have

$$e^{A_1}=S_1+S_2,\quad e^{A_2}=S_1-S_2,\quad e^{2H}=S_1^2+S_2^2$$

$$e^{2B_j}=S_1^2+S_2^2+2S_1S_2\cos\frac{j\pi}{2m},\quad e^{2D_j}=S_1^2+S_2^2-2S_1S_2\cos\frac{j\pi}{2m} \tag{8}$$

Example [2]. Let $4m=12$. From (3) we have

$$A_1=(t_1+t_{11})+(t_2+t_{10})+(t_3+t_9)+(t_4+t_8)+(t_5+t_7)+t_6,$$

$$A_2=-(t_1+t_{11})+(t_2+t_{10})-(t_3+t_9)+(t_4+t_8)-(t_5+t_7)+t_6,$$

$$H=-(t_2+t_{10})+(t_4+t_8)-t_6,$$

$$B_1=(t_1+t_{11})\cos\frac{\pi}{6}+(t_2+t_{10})\cos\frac{2\pi}{6}+(t_3+t_9)\cos\frac{3\pi}{6}+(t_4+t_8)\cos\frac{4\pi}{6}+(t_5+t_7)\cos\frac{5\pi}{6}-t_6,$$

$$B_2=(t_1+t_{11})\cos\frac{2\pi}{6}+(t_2+t_{10})\cos\frac{4\pi}{6}+(t_3+t_9)\cos\frac{6\pi}{6}+(t_4+t_8)\cos\frac{8\pi}{6}+(t_5+t_7)\cos\frac{10\pi}{6}+t_6,$$

$$D_1=-(t_1+t_{11})\cos\frac{\pi}{6}+(t_2+t_{10})\cos\frac{2\pi}{6}-(t_3+t_9)\cos\frac{3\pi}{6}+(t_4+t_8)\cos\frac{4\pi}{6}-(t_5+t_7)\cos\frac{5\pi}{6}-t_6,$$

$$D_2=-(t_1+t_{11})\cos\frac{2\pi}{6}+(t_2+t_{10})\cos\frac{4\pi}{6}-(t_3+t_9)\cos\frac{6\pi}{6}+(t_4+t_8)\cos\frac{8\pi}{6}-(t_5+t_7)\cos\frac{10\pi}{6}+t_6,$$

$$A_1+A_2+2(H+B_1+B_2+D_1+D_2)=0, \quad A_2+2B_2=3(-t_3+t_6-t_9). \qquad (9)$$

From (8) and (9) we have

$$\exp[A_1+A_2+2(H+B_1+B_2+D_1+D_2)]=S_1^{12}-S_2^{12}=(S_1^3)^4-(S_2^3)^4=1. \qquad (10)$$

From (9) we have

$$\exp(A_2+2B_2)=[\exp(-t_3+t_6-t_9)]^3. \qquad (11)$$

From (8) we have

$$\exp(A_2+2B_2)=(S_1-S_2)(S_1^2+S_2^2+S_1S_2)=S_1^3-S_2^3. \qquad (12)$$

From (11) and (12) we have Fermat's equation

$$\exp(A_2+2B_2)=S_1^3-S_2^3=[\exp(-t_3+t_6-t_9)]^3. \qquad (13)$$

Fermat proved that (10) has no rational solutions for exponent 4 [8].

Therefore we prove we prove that (13) has no rational solutions for exponent 3. [2]

Theorem . Let $4m=4P$, where P is an odd prime, $(P-1)/2$ is an even number.

From (3) and (8) we have

$$\exp[A_1+A_2+2H+2\sum_{j=1}^{P-1}(B_j+D_j)]=S_1^{4P}-S_2^{4P}=(S_1^P)^4-(S_2^P)^4=1. \qquad (14)$$

From (3) we have

$$\exp[A_2+2\sum_{j=1}^{\frac{P-1}{4}}(B_{4j-2}+D_{4j})]=[\exp(-t_P+t_{2P}-t_{3P})]^P. \qquad (15)$$

From (8) we have

$$\exp[A_2+2\sum_{j=1}^{\frac{P-1}{4}}(B_{4j-2}+D_{4j})]=S_1^P-S_2^P. \qquad (16)$$

From (15) and (16) we have Fermat's equation

$$\exp[A_2+2\sum_{j=1}^{\frac{P-1}{4}}(B_{4j-2}+D_{4j})]=S_1^P-S_2^P=[\exp(-t_P+t_{2P}-t_{3P})]^P. \qquad (17)$$

Fermat proved that (14) has no rational solutions for exponent 4 [8]. Therefor we prove that (17) has no rational solutions for prime exponent P.

Remark. Mathematicians said Fermat could not possibly had a proof, because they do not understand FLT.In complex hyperbolic functions let exponent n be $n=\Pi P$, $n=2\Pi P$ and $n=4\Pi P$. Every factor of exponent n has Fermat's equation [1-7]. Using modular elliptic curves Wiles and Taylor prove FLT [9,10]. This is not the proof that Fermat thought to have had. The classical theory of automorphic functions,created by Klein and Poincare, was concerned with the study of analytic functions in the unit circle that are invariant under a discrete group of transformation. Automorphic functions are the generalization of trigonometric, hyperbolic elliptic, and certain other functions of elementary analysis. The complex trigonometric functions and complex hyperbolic functions have a wide application in mathematics and physics.

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Riemann Paper (1859) Is False

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Abstract: In 1859 Riemann defined the zeta function $\zeta(s)$. From Gamma function he derived the zeta function with Gamma function $\bar{\zeta}(s)$. $\bar{\zeta}(s)$ and $\zeta(s)$ are the two different functions. It is false that $\bar{\zeta}(s)$ replaces $\zeta(s)$. After him later mathematicians put forward Riemann hypothesis(RH) which is false. The Jiang function $J_n(\omega)$ can replace RH.

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In 1859 Riemann defined the Riemann zeta function (RZF)[1]

$$\zeta(s)=\prod_{P}(1-P^{-s})^{-1}=\sum_{n=1}^{\infty}\frac{1}{n^{s}}, \qquad (1)$$

where $s=\sigma+ti, i=\sqrt{-1}$, σ and t are real, P ranges over all primes. RZF is the function of the complex variable s in $\sigma\geq 0, t\neq 0$, which is absolutely convergent.

In 1896 J. Hadamard and de la Vallee Poussin proved independently [2]

$$\zeta(1+ti)\neq 0. \qquad (2)$$

In 1998 Jiang proved [3]

$$\zeta(s) \neq 0, \tag{3}$$

where $0 \leq \sigma \leq 1$.

Riemann paper (1859) is false [1] We define Gamma function [1, 2]

$$\Gamma\left(\frac{s}{2}\right) = \int_0^\infty e^{-t} t^{\frac{s}{2}-1} dt. \tag{4}$$

For $\sigma > 0$. On setting $t = n^2 \pi x$, we observe that

$$\pi^{-\frac{s}{2}} \Gamma\left(\frac{s}{2}\right) n^{-s} = \int_0^\infty x^{\frac{s}{2}-1} e^{-n^2 \pi x} dx. \tag{5}$$

Hence, with some care on exchanging summation and integration, for $\sigma > 1$,

$$\pi^{-\frac{s}{2}} \Gamma\left(\frac{s}{2}\right) \overline{\varsigma}(s) = \int_0^\infty x^{\frac{s}{2}-1} \left(\sum_{n=1}^{\infty} e^{-n^2 \pi x} \right) dx$$

$$= \int_0^\infty x^{\frac{s}{2}-1} \left(\frac{\vartheta(x) - 1}{2} \right) dx, \tag{6}$$

where $\overline{\zeta}(s)$ is called Riemann zeta function with gamma function rather than $\zeta(s)$,

$$\vartheta(x) := \sum_{n=-\infty}^{\infty} e^{-n^2 \pi x}, \tag{7}$$

is the Jacobi theta function. The functional equation for $\vartheta(x)$ is

$$x^{\frac{1}{2}} \vartheta(x) = \vartheta(x^{-1}), \tag{8}$$

and is valid for $x > 0$.

Finally, using the functional equation of $\vartheta(x)$, we obtain

$$\overline{\zeta}(s) = \frac{\pi^{\frac{s}{2}}}{\Gamma\left(\frac{s}{2}\right)} \left\{ \frac{1}{s(s-1)} + \int_1^\infty (x^{\frac{s}{2}-1} + x^{-\frac{s}{2}-\frac{1}{2}}) \cdot (\frac{\vartheta(x) - 1}{2}) dx \right\}. \tag{9}$$

From (9) we obtain the functional equation

$$\pi^{-\frac{s}{2}} \Gamma\left(\frac{s}{2}\right) \overline{\zeta}(s) = \pi^{-\frac{1-s}{2}} \Gamma\left(\frac{1-s}{2}\right) \overline{\zeta}(1-s). \tag{10}$$

The function $\overline{\zeta}(s)$ satisfies the following

1. $\overline{\zeta}(s)$ has no zero for $\sigma > 1$;
2. The only pole of $\overline{\zeta}(s)$ is at $s = 1$; it has residue 1 and is simple;
3. $\overline{\zeta}(s)$ has trivial zeros at $s = -2, -4, \ldots$ but $\zeta(s)$ has no zeros;
4. The nontrivial zeros lie inside the region $0 \leq \sigma \leq 1$ and are symmetric about both the vertical line $\sigma = 1/2$.

The strip $0 \leq \sigma \leq 1$ is called the critical strip and the vertical line $\sigma = 1/2$ is called the critical line.

Conjecture (The Riemann Hypothesis). All nontrivial zeros of $\overline{\zeta}(s)$ lie on the critical line $\sigma = 1/2$, which is false. [3]

$\overline{\zeta}(s)$ and $\zeta(s)$ are the two different functions. It is false that $\overline{\zeta}(s)$ replaces $\zeta(s)$, Pati proved that is not all complex zeros of $\overline{\zeta}(s)$ lie on the critical line: $\sigma = 1/2$ [4].

Schadeck pointed out that the falsity of RH implies the falsity of RH for finite fields [5, 6]. RH is not directly related to prime theory. Using RH mathematicians prove many prime theorems which is false. In 1994 Jiang discovered Jiang function $J_n(\omega)$ which can replace RH, Riemann zeta function and L-function in view of its proved feature: if $J_n(\omega) \neq 0$ then the prime equation has infinitely many prime solutions; and if $J_n(\omega) = 0$, then the prime equation has finitely many prime solutions. By using $J_n(\omega)$ Jiang proves about 600 prime theorems including the Goldbach's theorem, twin prime theorem and theorem on arithmetic progressions in primes[7,8].

In the same way we have a general formula involving $\overline{\zeta}(s)$

$$\int_0^\infty x^{s-1}\sum_{n=1}^\infty F(nx)dx = \sum_{n=1}^\infty \int_0^\infty x^{s-1}F(nx)dx$$

$$= \sum_{n=1}^\infty \frac{1}{n^s}\int_0^\infty y^{s-1}F(y)dy = \overline{\zeta}(s)\int_0^\infty y^{s-1}F(y)dy, \qquad (11)$$

where $F(y)$ is arbitrary.

From (11) we obtain many zeta functions $\overline{\zeta}(s)$ which are not directly related to the number theory.

The prime distributions are order rather than random. The arithmetic progressions in primes are not directly related to ergodic theory ,harmonic analysis, discrete geometry, and combinatories. Using the ergodic theory Green and Tao prove that there exist infinitely many arithmetic progressions of length k consisting only of primes which is false [9, 10, 11]. Fermat's last theorem (FLT) is not directly related to elliptic curves. In 1994 using elliptic curves Wiles proved FLT which is false [12]. There are Pythagorean theorem and FLT in the complex hyperbolic functions and complex trigonometric functions. In 1991 without using any number theory Jiang proved FLT which is Fermat's marvelous proof[7, 13].

Primes Represented by $P_1^n + mP_2^n$ [14]

（1）Let $n=3$ and $m=2$. We have

$$P_3 = P_1^3 + 2P_2^3.$$

We have Jiang function

$$J_3(\omega) = \prod_{3\le P}(P^2 - 3P + 3 - \chi(P)) \neq 0,$$

Where $\chi(P) = 2P-1$ if $2^{\frac{P-1}{3}} \equiv 1 \pmod P$; $\chi(P) = -P+2$ if $2^{\frac{P-1}{3}} \not\equiv 1 \pmod P$; $\chi(P) = 1$ otherwise.

Since $J_n(\omega) \neq 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is a prime.

We have the best asymptotic formula

$$\pi_2(N,3) = \left|\{P_1, P_2 : P_1, P_2 \le N, P_1^3 + 2P_2^3 = P_3 \text{ prime}\}\right|$$

$$\sim \frac{J_3(\omega)\omega}{6\Phi^3(\omega)}\frac{N^2}{\log^3 N} = \frac{1}{3}\prod_{3\le P}\frac{P(P^2-3P+3-\chi(P))}{(P-1)^3}\frac{N^2}{\log^3 N}.$$

where $\omega = \prod_{2\le P} P$ is called primorial, $\Phi(\omega) = \prod_{2\le P}(P-1)$.

It is the simplest theorem which is called the Heath-Brown problem [15].

（2）Let $n=P_0$ be an odd prime, $2|m$ and $m\neq\pm b^{P_0}$.
we have
$P_3=P_1^{P_0}+mP_2^{P_0}$
We have
$$J_3(\omega)=\prod_{3\le P}(P^2-3P+3-\chi(P))\neq 0,$$
where $\chi(P)=-P+2$ if $P|m$; $\chi(P)=(P_0-1)P-P_0+2$ if $m^{\frac{P-1}{P_0}}\equiv 1$ (mod P);
$\chi(P)=-P+2$ if $m^{\frac{P-1}{P_0}}\not\equiv 1$ (mod P); $\chi(P)=1$ otherwise.
Since $J_n(\omega)\neq 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is a prime.
We have
$$\pi_2(N,3)\sim\frac{J_3(\omega)\omega}{2P_0\Phi^3(\omega)}\frac{N^2}{\log^3 N}.$$

The Polynomial $P_1^n+(P_2+1)^2$ Captures Its Primes [14]

（1）Let $n=4$, We have
$P_3=P_1^4+(P_2+1)^2$,
We have Jiang function
$$J_3(\omega)=\prod_{3\le P}(P^2-3P+3-\chi(P))\neq 0,$$
Where $\chi(P)=P$ if $P\equiv 1$ (mod 4); $\chi(P)=P-4$ if $P\equiv 1$（mod 8）; $\chi(P)=-P+2$ otherwise.
Since $J_n(\omega)\neq 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is a prime.
We have the best asymptotic formula
$$\pi_2(N,3)=\left|\{P_1,P_2:P_1,P_2\le N,P_1^4+(P_2+1)^2=P_3\text{ prime}\}\right|$$
$$\sim\frac{J_3(\omega)\omega}{8\Phi^3(\omega)}\frac{N^2}{\log^3 N}.$$
It is the simplest theorem which is called Friedlander-Iwaniec problem [16].

（2）Let $n=4m$, We have
$P_3=P_1^{4m}+(P_2+1)^2$,
where $m=1,2,3,\cdots$.
We have Jiang function
$$J_3(\omega)=\prod_{3\le P\le P_i}(P^2-3P+3-\chi(P))\neq 0,$$
where $\chi(P)=P-4m$ if $8m|(P-1)$; $\chi(P)=P-4$ if $8|(P-1)$; $\chi(P)=P$ if $4|(P-1)$; $\chi(P)=-P+2$ otherwise.

Since $J_3(\omega)\neq 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is a prime. It is a generalization of Euler proof for the existence of infinitely many primes.

We have the best asymptotic formula

$$\pi_2(N,3) \sim \frac{J_3(\omega)\omega}{8m\Phi^3(\omega)}\frac{N^2}{\log^3 N}.$$

（3）Let $n=2b$. We have

$$P_3 = P_1^{2b} + (P_2+1)^2,$$

where b is an odd.

We have Jang function

$$J_3(\omega) = \prod_{3\le P}(P^2-3P+3-\chi(P)) \neq 0,$$

Where $\chi(P)=P-2b$ if $4b|(P-1)$; $\chi(P)=P-2$ if $4|(P-1)$; $\chi(P)=-P+2$ otherwise.

We have the best asymptotic formula

$$\pi_2(N,3) \sim \frac{J_3(\omega)\omega}{4b\Phi^3(\omega)}\frac{N^2}{\log^3 N}.$$

（4）Let $n=P_0$, We have

$$P_3 = P_1^{P_0} + (P_2+1)^2.$$

where P_0 is an odd. Prime.

we have Jiang function

$$J_3(\omega) = \prod_{3\le P}(P^2-3P+3-\chi(P)) \neq 0,$$

where $\chi(P)=P_0+1$ if $P_0|(P-1)$; $\chi(P)=0$ otherwise.

Since $J_3(\omega)\neq 0$, there exist infinitely many primes P_1 and P_2 such that P_3 is also a prime.

We have the best asymptotic formula

$$\pi_2(N,3) \sim \frac{J_3(\omega)\omega}{2P_0\Phi^3(\omega)}\frac{N^2}{\log^3 N}.$$

The Jiang function $J_n(\omega)$ is closely related to the prime distribution. Using $J_n(\omega)$ we are able to tackle almost all prime problems in the prime distributions.

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