The New Prime theorem (18)

Hardy-Littlewood Conjecture $E: x^2+1$

Chun-Xuan Jiang

P. O. Box 3924, Beijing 100854, P. R. China
jiangchunxuan@vip.sohu.com

Abstract: Using Jiang function we prove Hardy-Littlewood conjecture $E: x^2+1$ [2].

[Chun-Xuan Jiang. **The New Prime theorem (18) Hardy-Littlewood Conjecture** $E: x^2+1$**.** *Academ Arena* 2015;7(1s): 26-27]. (ISSN 1553-992X). http://www.sciencepub.net/academia. 18

Keywords: prime; theorem; function; number; new

Theorem. The prime equation

$$P_n = (2P_1P_2\cdots P_{n-1})^2+1 \tag{1}$$

has infinitely many prime solutions.

Proof. We have Jiang function[1]

$$J_n(\omega)=\prod_P[(P-1)^{n-1}-\chi(P)], \tag{2}$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$(2q_1q_2\cdots q_{n-1})^2+1\equiv 0 \pmod{P}, \quad q_i=1,\cdots,P-1, i=1,\cdots,n-1, \tag{3}$$

From (3) we have

$\left(\frac{-1}{P}\right)=(-1)^{\frac{P-1}{2}}$, if $\left(\frac{-1}{P}\right)=1$ then $\chi(P)=2(P-1)^{n-2}$, if $\left(\frac{-1}{P}\right)=-1$ then $\chi(P)=0$.

Substituting it into (2) we have.

$$J_n(\omega)=\prod_{3\le P}[(P-1)^{n-2}(P-2-(-1)^{\frac{P-1}{2}}]\neq 0. \tag{4}$$

We prove that (1) has infinitely many prime solutions. $J_n(\omega)\subset\phi^{n-1}(\omega)$

We have the best asymptotic formula [1]

$$\pi_2(N,n)=\left|\{P_1,\cdots,P_{n-1}\le N: P_n=prime\}\right|\sim\frac{J_n(\omega)\omega}{2\times(n-1)!\phi^n(\omega)}\frac{N^{n-1}}{\log^n N}. \tag{5}$$

Example 1. Let $n=2$. From (1) we have

$$P_2=(2P_1)^2+1 \tag{6}$$

From (4) we have

$$J_2(\omega)=\prod_{3\le P}[P-2-(-1)^{\frac{P-1}{2}}]\neq 0 \tag{7}$$

Example 2. Let $n=3$. From (1) we have

$$P_3=(2P_1P_2)^2+1. \tag{8}$$

From (4) we have

$$J_3(\omega)=\prod_{3\le P}[(P-1)(P-2-(-1)^{\frac{P-1}{2}}]\neq 0. \tag{9}$$

Note. The prime numbers theory is to count the Jiang function $J_{n+1}(\omega)$ and Jiang singular series $\sigma(J)=\frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)}=\prod_P\left(1-\frac{1+\chi(P)}{P}\right)(1-\frac{1}{P})^{-k}$ [1], which can count the number of prime number. The prime number is not random. But Hardy singular series $\sigma(H)=\prod_P\left(1-\frac{\nu(P)}{P}\right)(1-\frac{1}{P})^{-k}$ is false [2], which cannot count the number of prime numbers.

Author in US address:
Chun-Xuan Jiang
Institute for Basic Research Palm Harbor, FL 34682, U.S.A.
Jiangchunxuan@vip.sohu.com

References

1. Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. http://www. wbabin.net/math /xuan2. pdf. http://wbabin.net/xuan.htm#chun-xuan.
2. G. H. Hardy and J. E. Littlewood, Some problems of "Partitio Numerorum", III: On the expression of a number as a sum of primes. Acta Math., 44(1923)1-70.
3. Vinoo Cameron. **Prime Number 19, The Vedic Zero And The Fall Of Western Mathematics By Theorem.** *Nat Sci* 2013;11(2):51-52. (ISSN: 1545-0740). http://www.sciencepub.net/nature/ns1102/009_15631ns1102_51_52.pdf.
4. Vinoo Cameron, Theo Den otter. **PRIME NUMBER COORDINATES AND CALCULUS.** *Rep Opinion* 2012;4(10):16-17. (ISSN: 1553-9873). http://www.sciencepub.net/report/report0410/004_10859report0410_16_17.pdf.
5. Vinoo Cameron, Theo Den otter. **PRIME NUMBER COORDINATES AND CALCULUS.** *J Am Sci* 2012;8(10):9-10. (ISSN: 1545-1003). http://www.jofamericanscience.org/journals/am-sci/am0810/002_10859bam0810_9_10.pdf.
6. Chun-Xuan Jiang. **Automorphic Functions And Fermat's Last Theorem (1).** *Rep Opinion* 2012;4(8):1-6. (ISSN: 1553-9873). http://www.sciencepub.net/report/report0408/001_10009report0408_1_6.pdf.
7. Chun-Xuan Jiang. **Jiang's function $J_{n+1}(\omega)$ in prime distribution.** *Rep Opinion* 2012;4(8):28-34. (ISSN: 1553-9873). http://www.sciencepub.net/report/report0408/007_10015report0408_28_34.pdf.
8. Chun-Xuan Jiang**. The Hardy-Littlewood prime *k*-tuple conjecture is false.** *Rep Opinion* 2012;4(8):35-38. (ISSN: 1553-9873). http://www.sciencepub.net/report/report0408/008_10016report0408_35_38.pdf.
9. Chun-Xuan Jiang. **A New Universe Model.** *Academ Arena* 2012;4(7):12-13 (ISSN 1553-992X). http://sciencepub.net/academia/aa0407/003_10067aa0407_12_13.pdf.

5/1/2015