

**The New Prime theorem (17)**

$$P_n = 2P_1P_2 \cdots P_{n-1} \pm 1$$

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**Abstract:** Using Jiang function we prove that such that  $P_n = 2P_1P_2 \cdots P_{n-1} \pm 1$  has infinitely many prime solutions.

[Chun-Xuan Jiang. **The New Prime theorem (17)**  $P_n = 2P_1P_2 \cdots P_{n-1} \pm 1$ . *Academ Arena* 2015;7(1s): 24-25]. (ISSN 1553-992X). <http://www.sciencepub.net/academia>. 17

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**Theorem.** The prime equation

$$P_n = 2P_1P_2 \cdots P_{n-1} + 1 \quad (1)$$

has infinitely many prime solutions.

**Proof.** We have Jiang function[1]

$$J_n(\omega) = \prod_P [(P-1)^{n-1} - \chi(P)] \quad (2)$$

where  $\omega = \prod_P P$ ,  $\chi(P)$  is the number of solutions of congruence

$$2q_1q_2 \cdots q_{n-1} + 1 \equiv 0 \pmod{P}, \quad q_i = 1, \cdots, P-1, i = 1, \cdots, n-1 \quad (3)$$

From (3) we have

$$\chi(P) = (P-1)^{n-2} \quad (4)$$

Substituting (4) into (2) we have

$$J_n(\omega) = \prod_{3 \leq P} [(P-1)^{n-2}(P-2)] \neq 0 \quad (5)$$

We prove that (1) has infinitely many prime solutions.  $J_n(\omega) \subset \phi^{n-1}(\omega)$ .  
We have the best asymptotic formula [1]

$$\pi_2(N, n) = \left| \{P_1, \cdots, P_{n-1} \leq N : P_n = \text{prime}\} \right| \sim \frac{J_n(\omega)\omega}{(n-1)!\phi^n(\omega) \log^n N} \frac{N^{n-1}}{\log^n N} \quad (6)$$

Example 1. Let  $n = 2$ . From (1) we have

$$P_2 = 2P_1 + 1 \quad (7)$$

From (5) we have

$$J_2(\omega) = \prod_{3 \leq P} (P-2) \neq 0 \quad (8)$$

Example 2. Let  $n = 3$ . From (1) we have

$$P_3 = 2P_1P_2 + 1 \quad (9)$$

From (5) we have

$$J_3(\omega) = \prod_{3 \leq P} [(P-1)(P-2)] \neq 0 \quad (10)$$

In the same way we are able to prove that

$$P_m = 2P_1P_2 \cdots P_{n-1} - 1 \quad (11)$$

has infinitely many prime solutions.

Note. The prime numbers theory is to count the Jiang function  $J_{n+1}(\omega)$  and Jiang singular series  $\sigma(J) = \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)} = \prod_P \left(1 - \frac{1 + \chi(P)}{P}\right) \left(1 - \frac{1}{P}\right)^{-k}$  [1], which can count the number of prime number. The

prime number is not random. But Hardy singular series  $\sigma(H) = \prod_P \left(1 - \frac{\nu(P)}{P}\right) \left(1 - \frac{1}{P}\right)^{-k}$  is false [2-5], which cannot count the number of prime numbers.

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#### References

1. Chun-Xuan Jiang, Jiang's function  $J_{n+1}(\omega)$  in prime distribution. <http://www.wbabin.net/math/xuan2.pdf>. <http://wbabin.net/xuan.htm#chun0xuan>.
2. G. H. Hardy and J. E. Littlewood, Some problems of "Partitio Numerorum", III: On the expression of a number as a sum of primes. *Acta Math.*, 44(1923)-70.
3. B. Green and T. Tao, Linear equations in primes. To appear, *Ann. Math.*
4. D. Goldston, J. Pintz and C. Y. Yildirim, Primes in tuples I. *Ann. Math.*, 170(2009) 819-862.
5. Vinoo Cameron. **Prime Number 19, The Vedic Zero And The Fall Of Western Mathematics By Theorem.** *Nat Sci* 2013;11(2):51-52. (ISSN: 1545-0740). [http://www.sciencepub.net/nature/ns1102/009\\_15631ns1102\\_51\\_52.pdf](http://www.sciencepub.net/nature/ns1102/009_15631ns1102_51_52.pdf).
6. Vinoo Cameron, Theo Den otter. **PRIME NUMBER COORDINATES AND CALCULUS.** *Rep Opinion* 2012;4(10):16-17. (ISSN: 1553-9873). [http://www.sciencepub.net/report/report0410/004\\_10859report0410\\_16\\_17.pdf](http://www.sciencepub.net/report/report0410/004_10859report0410_16_17.pdf).
7. Vinoo Cameron, Theo Den otter. **PRIME NUMBER COORDINATES AND CALCULUS.** *J Am Sci* 2012;8(10):9-10. (ISSN: 1545-1003). [http://www.jofamericanscience.org/journals/am-sci/am0810/002\\_10859bam0810\\_9\\_10.pdf](http://www.jofamericanscience.org/journals/am-sci/am0810/002_10859bam0810_9_10.pdf).
8. Chun-Xuan Jiang. **Automorphic Functions And Fermat's Last Theorem (1).** *Rep Opinion* 2012;4(8):1-6. (ISSN: 1553-9873). [http://www.sciencepub.net/report/report0408/001\\_10009report0408\\_1\\_6.pdf](http://www.sciencepub.net/report/report0408/001_10009report0408_1_6.pdf).
9. Chun-Xuan Jiang. **Jiang's function  $J_{n+1}(\omega)$  in prime distribution.** *Rep Opinion* 2012;4(8):28-34. (ISSN: 1553-9873). [http://www.sciencepub.net/report/report0408/007\\_10015report0408\\_28\\_34.pdf](http://www.sciencepub.net/report/report0408/007_10015report0408_28_34.pdf).
10. Chun-Xuan Jiang. **The Hardy-Littlewood prime k-tuple conjecture is false.** *Rep Opinion* 2012;4(8):35-38. (ISSN: 1553-9873). [http://www.sciencepub.net/report/report0408/008\\_10016report0408\\_35\\_38.pdf](http://www.sciencepub.net/report/report0408/008_10016report0408_35_38.pdf).
11. Chun-Xuan Jiang. **A New Universe Model.** *Academ Arena* 2012;4(7):12-13 (ISSN 1553-992X). [http://sciencepub.net/academia/aa0407/003\\_10067aa0407\\_12\\_13.pdf](http://sciencepub.net/academia/aa0407/003_10067aa0407_12_13.pdf).

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