The New Prime theorem（16）： $P_j = (j)^n P + (k-j)^n,\ j = 1,\cdots,k-1$

Chun-Xuan Jiang

P. O. Box 3924, Beijing 100854, P. R. China. jiangchunxuan@vip.sohu.com

Abstract: Using Jiang function we prove that there exist infinitely many primes P such that each of $(j)^n P + (k-j)^n$ is a prime. [Chun-Xuan Jiang. **The New Prime theorem（16）** $P_j = (j)^n P + (k-j)^n,\ j = 1,\cdots,k-1$. *Academ Arena* 2015;7(1s): 23-23]. (ISSN 1553-992X). http://www.sciencepub.net/academia. 16

Keywords: prime; theorem; function; number; new

Theorem. Let k be a given prime.

$$P_j = (j)^n P + (k-j)^n (j = 1,\cdots,k-1, n = 1,2,\cdots) \tag{1}$$

There exist infinitely many prime P such that each of $(j)^n P + (k-j)^n$ is a prime.

Proof. We have Jiang function[1]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)], \tag{2}$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [(j)^n q + (k-j)^n] \equiv 0 \quad (\bmod P), q = 1,\cdots,P-1. \tag{3}$$

From (3) we have $\chi(2) = 0$, if $P < k$ then $\chi(P) \le P - 2, \chi(k) = 1$, if $k < P$ then $\chi(P) \le k - 1$. From (3) we have

$$J_2(\omega) \ne 0. \tag{4}$$

We prove that there exist inifinitely many primes P such that each of $(j)^n P + (k-j)^n$ is a prime.

Jiang function is a subset of Euler function: $J_2(\omega) \subset \phi(\omega)$. We have asymptotic formula

$$\pi_k(N,2) = \left|\{P \le N : (j)^n P + (k-j)^n = prime\}\right| \sim \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)} \frac{N}{\log^k N}. \tag{5}$$

where $\phi(\omega) = \prod_P (P-1)$.

Example 1. Let $k = 3$. From (1) we have

$$P_1 = P + 2^n, \quad P_2 = 2^n P + 1 \tag{6}$$

We have Jiang function

$$J_2(\omega) = \prod_{5 \le P} (P-3) \ne 0 \tag{7}$$

We prove that there exist infinitely many primes P such that P_1 and P_2 are all prime.

Reference

1. Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. http://www. wbabin.net/math /xuan2. pdf. http://wbabin.net/xuan.htm#chun-xuan.
2. Vinoo Cameron. **Prime Number 19, The Vedic Zero And The Fall Of Western Mathematics By Theorem.** *Nat Sci* 2013;11(2):51-52. (ISSN: 1545-0740). http://www.sciencepub.net/nature/ns1102/009_15631ns1102_51_52.pdf.
3. Vinoo Cameron, Theo Den otter. **PRIME NUMBER COORDINATES AND CALCULUS.** *Rep Opinion* 2012;4(10):16-17. (ISSN: 1553-9873). http://www.sciencepub.net/report/report0410/004_10859report0410_16_17.pdf.
4. Vinoo Cameron, Theo Den otter. **PRIME NUMBER COORDINATES AND CALCULUS.** *J Am Sci* 2012;8(10):9-10. (ISSN: 1545-1003). http://www.jofamericanscience.org/journals/am-sci/am0810/002_10859bam0810_9_10.pdf.
5. Chun-Xuan Jiang. **Automorphic Functions And Fermat's Last Theorem (1).** *Rep Opinion* 2012;4(8):1-6. (ISSN: 1553-9873).
6. Chun-Xuan Jiang. **A New Universe Model.** *Academ Arena* 2012;4(7):12-13 (ISSN 1553-992X).

5/1/2015