

The New Prime theorem (14)

$$P_j = (j)^2 P + (k - j)^2, j = 1, \dots, k - 1$$

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Abstract: Using Jiang function we prove that there exist infinitely many primes P such that each of $(j)^2 P + (k - j)^2$ is a prime.

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Theorem. Let k be a given prime.

$$P_j = (j)^2 P + (k - j)^2 (j = 1, \dots, k - 1) \quad (1)$$

There exist infinitely many prime P such that each of $(j)^2 P + (k - j)^2$ is a prime.

Proof. We have Jiang function[1]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \quad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1} [(j)^2 q + (h - j)^2] \equiv 0 \pmod{P}, q = 1, \dots, P - 1. \quad (3)$$

From (3) we have $\chi(2) = 0$ if $P < k$ then $\chi(P) \leq P - 2$, $\chi(k) = 1$, if $k < P$ then $\chi(P) \leq k - 1$.

Jiang functions a subset of Euler function: $J_2(\omega) \subset \phi(\omega)$. From (3) we have

$$J_2(\omega) \neq 0 \quad (4)$$

We prove that there exist infinitely many primes P such that each of $(j)^2 P + (k - j)^2$ is a prime.

We have asymptotic formula

$$\pi_k(N, 2) = \left| \left\{ P \leq N : (j)^2 P + (k - j)^2 = \text{prime} \right\} \right| \sim \frac{J_2(\omega) \omega^{k-1}}{\phi^k(\omega)} \frac{N}{\log^k N}, \quad (5)$$

where $\phi(\omega) = \prod_P (P - 1)$

We have [2]

$$\left| \left\{ P \leq N : jP + k - j = \text{prime} \right\} \right| \leq \left| \left\{ P \leq N : (j)^2 P + (k - j)^2 = \text{prime} \right\} \right| \quad (6)$$

Example 1. Let $K = 3$. From (1) we have

$$P_1 = P + 4, P_2 = 4P + 1 \quad (7)$$

We have Jiang function

$$J_2(\omega) = \prod_{5 \leq P} (P - 3) \neq 0 \quad (8)$$

There exist infinitely many primes P such that P_1 and P_2 are all prime.

We have asymptotic formula

$$\pi_3(N, 2) = |\{P \leq N : P_1 = \text{prime}, P_2 = \text{prime}\}| \sim \frac{J_2(\omega)\omega^2}{\phi^3(\omega)} \frac{N}{\log^3 N} \quad (9)$$

Example 2. Let $k = 5$, from (1) we have

$$P_j = (j)^2 P + (5 - j)^2 \quad (j = 1, 2, 3, 4) \quad (10)$$

We have jiang function

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \quad (11)$$

We have $\chi(3) = 1$, $\chi(5) = 1$, $\chi(7) = 2$, $\chi(11) = 2$, $\chi(13) = 3$, $\chi(17) = 3$, $\chi(P) = 4$ otherwise.

Substituting it into (11) we have

$$J_2(\omega) = 11232 \prod_{19 \leq P} (P - 5) \neq 0 \quad (12)$$

There exist infinitely many primes P such that P_1, P_2, P_3 and P_4 are all prime. We have asymptotic formula

$$\pi_5(N, 2) = |\{P \leq N : P_1, P_2, P_3, P_4 = \text{prime}\}| \sim \frac{J_2(\omega)\omega^4}{\phi^5(\omega)} \frac{N}{\log^5 N} \quad (13)$$

Example 3. Let $k = 7$. From (1) we have

$$P_j = (j)^2 P + (7 - j)^2 \quad (j = 1, 2, 3, 4, 5, 6) \quad (14)$$

We have jiang function

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)] \quad (15)$$

Where $\chi(2) = 0$, $\chi(3) = 1$, $\chi(5) = 2$, $\chi(7) = 1$, $\chi(11) = 5$, $\chi(13) = 5$, $\chi(17) = 4$, $\chi(29) = 5$, $\chi(37) = 5$, $\chi(P) = 6$ otherwise.

From (15) we have

$$J_2(\omega) \neq 0 \quad (16)$$

We prove that there exist infinitely many primes P such that each of $(j)^2 P + (7 - j)^2$ is a prime.

Note. The prime numbers theory is to count the Jiang function $J_{n+1}(\omega)$ and Jiang singular series

$$\sigma(J) = \frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)} = \prod_P \left(1 - \frac{1 + \chi(P)}{P}\right) \left(1 - \frac{1}{P}\right)^{-k} \quad [1-3], \text{ which can count the number of prime numbers. The}$$

$$\sigma(H) = \prod_P \left(1 - \frac{\nu(P)}{P}\right) \left(1 - \frac{1}{P}\right)^{-k}$$

prime number is not random. But Hardy singular series can not count the number of prime numbers.

is false [4-6], which

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