The New Prime theorem（13）
$n\times a^n\pm 1$ and $n\times 2^n\pm 1$

Chun-Xuan Jiang

P. O. Box 3924, Beijing 100854, P. R. China
jiangchunxuan@vip.sohu.com

Abstract: Using Jiang function we prove that $n\times a^n\pm 1$ have infinitely many prime solutions and $n\times 2^n\pm 1$ have finite prime solutions.

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Theorem. We define the irreducible prime equation

$$P_1=n\times(P-1)^n+1 \tag{1}$$

For every positive integer n there exist infinitely many primes P such that P_1 is a prime.

Proof. We have Jiang function[1]

$$J_2(\omega)=\prod_P[P-1-\chi(P)], \tag{2}$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$n\times(q-1)^n+1\equiv 0 \pmod P, \quad q=1,\cdots,P-1. \tag{3}$$

From (3) we have that if $n=3b+2$ then $\chi(3)=1$, $\chi(3)=0$ otherwise, $\chi(P)<P-1$. We have

$$J_2(\omega)\neq 0. \tag{4}$$

We prove that there exist infinitely many primes P such that P_2 is a prime.
We have asymptotic formula [1]

$$\pi_2(N,2)=\left|\left\{P\le N: n\times(P-1)^n+1=prime\right\}\right|\sim\frac{J_2(\omega)\omega}{n\phi^2(\omega)}\frac{N}{\log^2 N} \tag{5}$$

where $\phi(\omega)=\prod_P(P-1)$.

Let $P=3$. From (1) we have Cullen equation

$$P_1=n\times 2^n+1 \tag{6}$$

From (5) we have

$$\pi_2(3,2)\sim\frac{J_2(\omega)}{n\phi^2(\omega)}\frac{3}{\log^2 3}\to 0 \quad \text{as} \quad n\to\infty \tag{7}$$

We prove the finite Cullen primes.

In the same way we are able to prove that $n\times a^n-1$ has infinitely many prime solutions, $n\times 2^n-1$ has definite prime solutions and $h\times 2^n\pm 1$ have finite prime solutions.

Author in US address:
Chun-Xuan Jiang
Jiangchunxuan@vip.sohu.com
Institute for Basic Research Palm Harbor, FL 34682, U.S.A.

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