The New Prime theorem（11）
$$2 \times a^2 \pm 1$$

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Abstract: Using Jiang function we prove that $2 \times a^2 \pm 1$ has infinitely many prime solutions.

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Theorem. We define the prime equation

$$P_1 = 2 \times (P-1)^2 - 1 \qquad\qquad (1)$$

There exist infinitely many primes P such that P_2 is a prime.

Proof. We have Jiang function[1]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)], \qquad\qquad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$2 \times (q-1)^2 - 1 \equiv 0 \pmod{P}, \quad q = 1, \cdots, P-1 \qquad\qquad (3)$$

From (3) we have $(\dfrac{2}{P}) = (-)^{\frac{P^2-1}{8}}$, if $\left(\dfrac{2}{P}\right) = 1$ then $\chi(P) = 2$, if $\left(\dfrac{2}{P}\right) = -1$ then $\chi(P) = 0$.

Substituting it into (2) we have

$$J_2(\omega) = \prod_{3 \le P} \left[P - 2 - (-1)^{\frac{P^2-1}{8}} \right] \neq 0 \qquad\qquad (4)$$

We prove there exist infinitely many primes P such that P_2 is a prime.

We have asymptotic formula [1]:

$$\pi_2(N,2) = \left| \left\{ P \le N : 2 \times (P-1)^2 - 1 = prime \right\} \right| \sim \frac{J_2(\omega)\omega}{2\phi^2(\omega)} \frac{N}{\log^2 N} \qquad\qquad (5)$$

where $\phi(\omega) = \prod_P (P-1)$.

In the same way we are able to prove that $2 \times a^2 + 1$ has infinitely many prime solutions.

Reference

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