There are finite Mersenne primes and There are finite repunits primes

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Abstract: Using Jiang function we prove the finite Mersenne primes and the finite repunits primes.
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Theorem. Suppose the prime equation

$$P_1 = \frac{(P-1)^{P_0} - 1}{P-2}. \quad (1)$$

where P_0 is a given prime.

There exist infinitely many primes P such that P_1 is a prime.

Proof. We have Jiang function[1]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)], \quad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\frac{(q-1)^{P_0} - 1}{q-2} \equiv 0 \pmod{P}, \quad q = 1, \cdots, P-1. \quad (3)$$

$\chi(P_0) = 1$, $\chi(P) = P_0 - 1$ if $P \equiv 1 \pmod{P_0}$, $\chi(P) = 0$ otherwise.

Since $J_2(\omega) \neq 0$, there exist infinitely many primes P such that P_1 is a prime.

We have the asymptotic formula [1]

$$\pi_2(N,2) = \left|\{P \le N : P_1 = prime\}\right| \sim \frac{1}{P_0 - 1} \frac{J_2(\omega)\omega}{\phi^2(\omega)} \frac{N}{\log^2 N}. \quad (4)$$

where $\phi(\omega) = \prod_P (P-1)$.

Let $P = 3$. From (1) we have equation of Mersenne numbers [2]

$$P_1 = 2^{P_0} - 1. \quad (5)$$

From (4) we have

$$\pi_2(3,2) = \left|\{3 \le N : 2^{P_0} - 1 = prime\}\right| \sim \frac{1}{P_0 - 1} \frac{J_2(\omega)\omega}{\phi^2(\omega)} \frac{3}{\log^2 3} \to 0 \text{ as } P_0 \to \infty \quad (6)$$

We prove the finite Mersenne primes.

Let $P = 11$. From (1) we have equation of repunits numbers [2]

$$P_1 = \frac{10^{P_0} - 1}{9}. \quad (7)$$

From (4) we have

$$\pi_{11}(11,2) = \left|\left\{11 \le N : \frac{10^{P_0} - 1}{9} = prime\right\}\right| \sim \frac{1}{P_0 - 1} \frac{J_2(\omega)\omega}{\phi^2(\omega)} \frac{11}{\log^2 11} \to 0$$

as $P_0 \to \infty$. （8）

We prove the finite repunits primes.

In the same way we are able to prove that $(a^{P_0}-1)/(a-1)$ with $a = 4, 6, 10, 12, \cdots,$ has the finite prime solutions.

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Reference

1. Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. http://www. wbabin.net/math /xuan2. pdf.
2. P. Ribenboim, The new book of prime number records, 3rd edition, spring-Verlag, New York, NY, 1995.
1. Chun-Xuan Jiang. **Automorphic Functions And Fermat's Last Theorem (1).** *Rep Opinion* 2012;4(8):1-6]. (ISSN: 1553-9873). http://www.sciencepub.net/report/report0408/001_10009report0408_1_6.pdf.
2. Chun-Xuan Jiang. **Jiang's function $J_{n+1}(\omega)$ in prime distribution.** *Rep Opinion* 2012;4(8):28-34]. (ISSN: 1553-9873). http://www.sciencepub.net/report/report0408/007_10015report0408_28_34.pdf.
3. Chun-Xuan Jiang**. The Hardy-Littlewood prime *k*-tuple conjecture is false.** *Rep Opinion* 2012;4(8):35-38]. (ISSN: 1553-9873). http://www.sciencepub.net/report/report0408/008_10016report0408_35_38.pdf.
4. Chun-Xuan Jiang. **A New Universe Model.** *Academ Arena* 2012;4(7):12-13] (ISSN 1553-992X). http://sciencepub.net/academia/aa0407/003_10067aa0407_12_13.pdf.

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