There are finite Fermat primes

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Abstract: Using Jiang function we prove the finite Fermat primes.

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Theorem. Suppose the prime equation

$$P_1=(P-1)^{2^n}+1. \tag{1}$$

There exist infinitely many primes P such that P_1 is a prime.

Proof. We have Jiang function[1]

$$J_2(\omega)=\prod_P[P-1-\chi(P)], \tag{2}$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$(q-1)^{2^n}+1\equiv 0 \pmod P, \quad q=1,\cdots,P-1. \tag{3}$$

From (3) we have $\chi(P)=2^n$ if $P\equiv 1 \pmod{2^{n+1}}$, $\chi(P)=0$ otherwise.

Since $J_2(\omega)\neq 0$, there exist infinitely many primes P such that P_1 is a prime.

We have the asymptotic formula [1]

$$\pi_2(N,2)=\left|\left\{P\le N:(P-1)^{2^n}+1=prime\right\}\right|\sim\frac{J_2(\omega)}{2^n\phi^2(\omega)}\frac{N}{\log^2 N}. \tag{4}$$

When $P=3$. From (1) we have the equation of Fermat number [2]

$$P_1=2^{2^n}+1 \tag{5}$$

From (4) we have

$$\pi_2(3,2)=\left|\left\{3\le N:2^{2^n}+1=prime\right\}\right|\sim\frac{J_2(\omega)}{2^n\phi^2(\omega)}\frac{3}{\log^2 3}\to 0 \quad \text{as } n\to\infty \tag{4}$$

From (4) we prove the finite Fermat primes.

In the same way we are able to prove that $4^{2^n}+1$ and $6^{2^n}+1$ have finite prime solutions [2]

Reference

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