

On The Prime theorem:

$$P, jP + 15 - j \quad (j = 1, 2, 4, 7, 8, 11, 13, 14)$$

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Abstract: Using Jiang function we prove that there exist infinitely many primes P such that each $jP + 15 - j$ is a prime.

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Theorem.

$$P, jP + 15 - j \quad (j = 1, 2, 4, 7, 8, 11, 13, 14) \quad (1)$$

There exist infinitely many primes P such that each of $jP + 15 - j$ is a prime.

Proof. We have Jiang function[1]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)], \quad (2)$$

$$\text{where } \omega = \prod_P P,$$

$\chi(P)$ is the number of solutions of congruence

$$\Pi(jq + 15 - j) \quad (j = 1, 2, 4, 7, 8, 11, 13, 14) \equiv 0 \pmod{P} \quad (3)$$

$$q = 1, \dots, P - 1$$

From (3) we have $\chi(2) = 0$, $\chi(3) = 1$, $\chi(5) = 1$, $\chi(7) = 3$, $\chi(11) = 5$, $\chi(13) = 5$, $\chi(P) = 8$ otherwise.

From (3) and (2) we have

$$J_2(\omega) = 315 \prod_{17 \leq P} (P - 9) \neq 0 \quad (4)$$

We prove that there exist infinitely many primes P such that $jP + 15 - j$ is a prime.

We have the best asymptotic formula [1]

$$\pi_9(N, 2) = \left| \left\{ P \leq N : jP + 15 - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega)\omega^8}{\phi^9(\omega)} \frac{N}{\log^9 N}, \quad (5)$$

$$\text{where } \phi(\omega) = \prod_P (P - 1)$$

Reference

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