

On The Prime Equations: $P, jP + k - j (j = 1, \dots, k-1)$

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Abstract: Using Jiang function we prove that there exist infinitely many primes P such that each $jP + k - j$ is a prime.

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Theorem. Let k be a given prime.

$$P, jP + k - j (j = 1, \dots, k-1) \quad (1)$$

There exist infinitely many primes P such that each of $jP + k - j$ is a prime.

Proof. We have Jiang function[1]

$$J_2(\omega) = \prod_P [P-1 - \chi(P)], \quad (2)$$

where

$$\omega = \prod_P P,$$

$\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k-1} (jq + k - j) \equiv 0 \pmod{P}, \quad q = 1, \dots, P-1 \quad (3)$$

From (3) we have $\chi(2) = 0$, if $P < k$ then $\chi(P) = P-2$, $\chi(k) = 1$, if $k < P$ then $\chi(P) = k-1$.
From (3) and (2) we have

$$J_2(\omega) = (k-2) \prod_{k < p} (P-k) \neq 0 \quad (4)$$

We prove that there exist infinitely many primes P such that each of $jP + k - j$ is a prime
We have the asymptotic formula [1]

$$\pi_k(N, 2) = \left| \left\{ P \leq N : jP + k - j = \text{prime} \right\} \right| \sim \frac{J_2(\omega) \omega^{k-1}}{\phi^k(\omega)} \frac{N}{\log^k N}, \quad (5)$$

where $\phi(\omega) = \prod_P (P-1)$

Reference

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