

On The Prime Equations: $P, jP + 7 - j$ ($j = 1, 2, 3, 4, 5, 6$)

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Abstract: Using Jiang function we prove that there exist infinitely many primes P such that each $jP + 7 - j$ is a prime.

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Theorem.

$$P, jP + 7 - j \quad (j = 1, 2, 3, 4, 5, 6) \quad (1)$$

There exist infinitely many primes P such that each of $jP + 7 - j$ is a prime.

Proof. We have Jiang function [1]

$$J_2(\omega) = \prod_P [P - 1 - \chi(P)], \quad (2)$$

where

$$\omega = \prod_P P,$$

$\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^6 (jq + 7 - j) \equiv 0 \pmod{P}, \quad q = 1, \dots, P-1. \quad (3)$$

From (3) we have $\chi(2) = 0$, $\chi(3) = 1$, $\chi(5) = 3$, $\chi(7) = 1$, $\chi(P) = 6$ otherwise.

From (3) and (2) we have

$$J_2(\omega) = 5 \prod_{11 \leq P} (P - 7) \neq 0 \quad (4)$$

We prove that there exist infinitely many primes P such that each of $jP + 7 - j$ is a prime.
We have the best asymptotic formula [1]

$$\pi_7(N, 2) = |\{P \leq N : jP + 7 - j = \text{prime}\}| \sim \frac{J_2(\omega)\omega^6}{\phi^7(\omega)} \frac{N}{\log^7 N} \quad (5)$$

where $\phi(\omega) = \prod_P (P - 1)$.

Reference

1. Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. <http://www.wbabin.net/math/xuan2.pdf>.
2. Chun-Xuan Jiang. **Automorphic Functions And Fermat's Last Theorem (1).** *Rep Opinion* 2012;4(8):1-6]. (ISSN: 1553-9873). http://www.sciencepub.net/report/report0408/001_10009report0408_1_6.pdf
3. Chun-Xuan Jiang. **Jiang's function $J_{n+1}(\omega)$ in prime distribution.** *Rep Opinion* 2012;4(8):28-34]. (ISSN: 1553-9873). http://www.sciencepub.net/report/report0408/007_10015report0408_28_34.pdf
4. Chun-Xuan Jiang. **The Hardy-Littlewood prime k -tuple conjecture is false.** *Rep Opinion* 2012;4(8):35-38]. (ISSN: 1553-9873). http://www.sciencepub.net/report/report0408/008_10016report0408_35_38.pdf
5. Chun-Xuan Jiang. **A New Universe Model.** *Academ Arena* 2012;4(7):12-13] (ISSN 1553-992X). http://sciencepub.net/academia/aa0407/003_10067aa0407_12_13.pdf