

Force exerted by Hawking radiation emitted from Black hole

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Abstract : Hawking radiation (also known as Bekenstein-Hawking radiation) is a thermal radiation with a black body spectrum predicted to be emitted by black holes due to quantum effects. The Hawking radiation process reduces the mass of the black hole and is therefore also known as black hole evaporation. Force exerted by hawking radiation is defined as function of entropy of black hole emitting hawking radiation ,density of black hole and schwarzschild radius of blackhole .The above equation $F = K \rho / r_s S$ (where F = Force exerted by hawking radiation , K=proportionality constant , r_s = schwarzschild radius of black hole ,s = entropy of black hole , ρ =black hole density) was developed based on quantum mechanical concepts . The above equation also describes outward force is exerted by hawking radiation to overcome the gravitational force of attraction of black hole .

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Key words : force,density,entropy

Introduction : When particles escape as thermal radiation, the black hole loses a small amount of its energy and therefore of its mass (mass and energy are related by Einstein's equation $E = mc^2$).The power emitted by a black hole in the form of Hawking radiation can easily be estimated for the simplest case of a nonrotating, non-charged Schwarzschild black hole of mass 'M' .Hawking radiation consist of photons, neutrinos, and to a lesser extent all sorts of massive particles.By the application of quantum mechanical concepts we can derive an equation for force is exerted by hawking radiation to overcome the gravitational force of attraction of black hole .

DERIVATION:

FORCE EXERTED BY HAWKING RADIATION EMITTED BY BLACK HOLE

When quantum mechanical effects are taken into account one finds that Blackhole emit thermal radiation (hawking radiation) at a temperature(hawking radiation temperature) is given by

$T = \frac{hc}{8\pi} \frac{GM}{k}$ where h=planck's constant, G=universal gravitational constant

M= Mass blackhole, k=Boltzmann constant, c=speed of light in vaccum /air

Schwarzschild radius of black hole can be given by $r_s = \frac{2GM}{c^2}$

Thus $T = \frac{hc}{8\pi} \frac{GM}{k}$ becomes $T = \frac{hc}{8\pi} \frac{c^2}{2k} \frac{2GM}{c^2}$ i.e $T = \frac{hc}{8\pi} \frac{2k}{rs}$

$KT = \frac{hc}{8\pi} \frac{2}{rs}$

According to **Boltzmann's law**: Energy of emitted thermal radiation by black hole is directly proportional to it's temperature given by $E=KT$ where k = Boltzmann constant

Then the equation $KT=hc/8\pi^2rs$ becomes $E=hc/8\pi^2rs$

Emitted thermal radiation by black hole will exert outward force to overcome the gravitational force of attraction of black hole. Hence energy of emitted thermal radiation can also be given by $E=F\lambda$ where E = energy of emitted thermal radiation, F =force exerted by radiation, λ =wavelength of emitted radiation.

(Proof for $E=F\lambda$ is shown at the end of derivation)

Thus $E=hc/8\pi^2rs$ becomes $F\lambda=hc/8\pi^2rs$

Debroglie wavelength associated with the emitted hawking radiation can be given by $\lambda=h/mc$

Where m = mass of emitted hawking radiation

Note: radiation travels at speed of light i.e c ($3*10^8$ m/s)

Thus $F\lambda/hc=hc/8\pi^2rs$ hence $F=mc^2/8\pi^2rs$ is obtained.

The **rate of energy** flow from black hole is given by $P=e\sigma T^4 A$.

Where P =rate of energy flow at temperature T , e =emissivity power(for black hole $e=1$)

A =surface area of black body. i.e $P=\sigma T^4 A$ is obtained.

According to **Stefan 's law** :Energy of emitted radiation from blackhole is directly proportional to fourth power of it's temperature $E = \sigma T^4$.where σ =stefan's constant.

Energy of emitted hawking radiation can be given by $E=mc^2$,where m =mass of emitted hawking radiation

By equivalence of **stefan's law** and **einstein's mass energy equivalence** law we get

$$mc^2 = \sigma T^4 \text{ then the equation } P = \sigma T^4 A \text{ becomes } P = mc^2 A.$$

then $P = mc^2 A$ i.e $P/A = mc^2$

Then the equation $F = mc^2/8\pi^2rs$ becomes $F = P/8\pi^2rs A$

where A =surface area of black hole emitting hawking radiation.

$$\text{Entropy of black hole emitting hawking radiation is given by } S = KA/4l_p^2$$

Where l_p = planck's length , S = entropy of black hole emitting hawking radiation

By rearranging the above equation we get $A = 4Sl_p^2/K$

i.e $F = P/8\pi^2rs A$ becomes $F = PK/8\pi^2rs 4Sl_p^2$ i.e $F = PK/32\pi^2rs Sl_p^2$.

Rate of rate of energy flow by blackhole is given by $P = \rho hG/180\pi$

where ρ =black hole density, G = universal gravitational constant.

then the equation $F = PK/32\pi^2rs Sl_p^2$ becomes $F = \rho hGK/180\pi(32\pi^2rs Sl_p^2)$

Planck's length is given by $l_p^2 = \frac{Gh}{2\pi c^3}$.

Then $F = \frac{\rho h G K}{180 \pi (32 \pi^2 r_s S l_p^2)}$ becomes $F = \frac{\rho h G K^2 \pi^3}{5760 \pi^3 r_s G h S}$

$$F = \frac{\rho K c^3}{2880 \pi^2 r_s S}$$

As $K' = \frac{K c^3}{2880 \pi^2}$, where K' = proportionality constant

Thus equation $F = \frac{K' \rho}{r_s S}$ is obtained. Where ρ = black hole density, r_s = Schwarzschild radius of black hole,

F = force exerted by hawking radiation, S = entropy of black hole

PROOF FOR THE EQUATION $E = F \lambda$

Determination of the Photon Force and Pressure

Reissig, Sergej

The 35th Meeting of the Division of Atomic, Molecular and Optical Physics, May 25-29, 2004, Tuscon, AZ.
MEETING ID: DAMOP04, abstract #D1.102

In [1] the formula for the practical determination of the power of a light particle was derived: $P = hf^2 (W)$ (1). For the praxis it is very usefully to define the forces and pressure of the electromagnetic or high temperature heat radiation. The use of the impulse equation $F = \frac{dP}{dt} = \frac{d(mc)}{dt}$ (2) together with the Einstein formula for $E = mc^2$ leads to the following relationship: $F = \frac{dE}{c dt} = \frac{dE}{c dt}$ (3) In [1] was shown: $F = \frac{dE}{c dt} = P$ (4). With the use the eq. (1), (3), (4) the force value could be finally determinated: $|F| = \frac{hf^2}{c}$ or $|F| = \frac{hc \lambda^{-2}}{c} = \frac{E \lambda}{N}$. The pressure of the photon could be calculated with using of the force value and effective area: $p = \frac{F A}{A} [Pa]$. References 1. About the calculation of the photon power. S. Reissig, APS four corners meeting, Arizona, 2003 -www.eps.org/aps/meet/4CF03/baps/abs/S150020.html

Note: Emitted hawking radiation also possess wavelength and energy during it's motion then it also exerts outward force to overcome the gravitational force of attraction of black hole.

Then the above equation $E = F \lambda$ can be applied to emitted hawking radiation also.

Result :

- 1) Force exerted by hawking radiation emitted by black hole as a function of black hole density, Schwarzschild radius of black hole, entropy of black hole emitting thermal radiation is given by $F = \frac{K' \rho}{r_s S}$

Discussions : Normally, a black hole is considered to draw all matter and energy in the surrounding region into it, as a result of the intense gravitational fields. Because Hawking radiation allows black holes to lose mass, black holes that lose more matter than they gain through other means are expected to dissipate, shrink, and ultimately vanish. Smaller micro black holes (MBHs) are predicted to be larger net emitters of radiation than larger black holes, and to shrink and dissipate faster. In order to overcome the gravitational force of attraction of black hole hawking radiation should posses outward force such that particles of thermal radiation are emitted from black hole.

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References :

- 1) CI Robert M. Wald, General Relativity, Sections 14.2-14.4, [University of Chicago Press](#), Chicago, 1984. (A good precise introduction to the subject.)
- 2) Hawking radiation (From [Wikipedia](#), the free encyclopedia)
- 3) Stephen W. Hawking, Particle creation by black holes, Commun. *Math. Phys.* 43 (1975), 199-220. (The original paper.)
- 4) Black hole thermodynamics (From [Wikipedia](#), the free encyclopedia)
- 5) Mass –energy equivalence, www.worsleyschool.net/science/files/emc2.html cached pages , (From [Wikipedia](#), the free encyclopedia).
- 6) Thermal radiation (From the free [Wikipedia](#) encyclopedia)
- 7) Blackhole (From the free [Wikipedia](#) encyclopedia)
- 8) Determination of the Photon Force and Pressure Reissig, Sergej ([www google.com](http://www.google.com))

Conclusion : According to the general theory of relativity, a black hole is a region of space from which nothing, including light, can escape. It is the result of the deformation of spacetime caused by a very compact mass. Around a black hole there is an undetectable surface which marks the point of no return, called an event horizon. It is called "black" because it absorbs all the light that comes towards it, reflecting nothing, just like a perfect black body in thermodynamics. Force is exerted by hawking radiation to overcome the gravitational force of attraction of black hole . As $F \propto \rho / r_s S$ if density of blackhole is more, then force exerted by hawking radiation is more ($F \propto \rho$). As schwarzschild radius of black hole is more, then force exerted by hawking radiation is less ($F \propto 1 / r_s$) .As entropy of black hole is more, then force exerted by hawking radiation is less ($F \propto 1 / S$).

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