Relationship between Continuity and Momentum Equation in Two Dimensional Flow

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Abstract

In this paper a quantitative discussion on a theory describing the relationship between the continuity and momentum equation in two dimensional flow together with the momentum equation in vectorial form: $\rho \, {}^{dq}\!/\!_{dt} = -\nabla p + \rho g + \mu \nabla^2 q$, on expanding $\nabla * (\nabla . q)$ in cylindrical polar coordinates, the end result proved to be Euler equation. [Academia Arena, 2009;1(1):62-72]. ISSN 1553-992X.

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Classification: Fluid Mechanics

1.0 Introduction:

A more detailed view of the fluxes across the parcel can be obtained within a reasonable space of text of we restrict our attention to two dimensions. We can then write the equations for the component and look closely at the change in these components.

We consider planar view of a parcel unit depth. Assume ρ is constant across the parcel, so we can write for the mass of the parcel, $\delta\mu = \rho\delta v = \rho\delta x\delta y\delta z$. In the two-dimensional flow, each component of velocity can vary in both x and y directions. We can approximate those velocity changes across our incremental parcel by a Taylor expansion. In this case we will consider the base values of qualities such as pressure and velocity to be the value of the center of the parcel and expand around these values. Note that value of the corner, $x = y = z = 0$, could also be assured as base values. Since the parcel is infinitesimal with respect to mean flow scales. The magnitudes of these values are uniform across the parcel in the limit $\delta v \to 0$. We are writing the incremental change at the point, we need not be zero. Again we look at the total change in the density and the scope of the parcel as it instantaneously occupies the point (x, y). We can derive the continuity equation in a slightly different manner, by considering a specific infinitesimal parcel in a Largrangian sense. The derivation will illustrate the close connection between Largrangian and Eulerian perspectives and we will send up with the familiar Eulerian expression. Starting with the Langrangian perspectives we consider a very small parcel such that $\delta v \to 0$, with no sinks or sources. We then follow the particular parcel that experiences volume and density changes with respects to five only field varcash will vary infinitesimal across the small dimensions of the parcel. Then the statement for the constant mass of fluids parcel, then the statement, for the constant mass of this parcel $\rho\delta v$ is completely expressed in the five derivatives, D(($\rho\delta v$)/ ∂t) $= 0$. However, when the parcel moves through the fluid, to volume must distorts and changes due to the changing forces in the thus field. The derivative which separated into density and volume changes by using the chain rule for differentiation. In the end, the derivative can be converted to the Eulerian expression.

2.0 Mathematical Analysis

The differences between the various derivatives can be explained in a more formal manner as follows:

Let consider a fluid particle moving with a load velocity;

q= $i\upsilon + \mathrm{j}\,v + \kappa\omega$ 1

and let the change of the property $b = b(x, y, z, t)$ of the particle be investigated. The change in b with time and position may be expressed as

$$\text{db}=\left(\frac{\partial b}{\partial t}\right)\partial t+\left(\frac{\partial b}{\partial x}\right)\partial x+\left(\frac{\partial b}{\partial y}\right)\partial y+\left(\frac{\partial b}{\partial z}\right)\partial z \qquad 2$$

The rate of change of s in time $\partial b / \partial t$ equation become

$$\frac{\nabla b}{\nabla t}=\frac{\partial b}{\partial t}+U\frac{\partial s}{\partial x}+V\frac{\partial b}{\partial y}+W\frac{\partial b}{\partial z}\} \qquad 3$$

$$\frac{\nabla b}{\nabla t}=\frac{\partial b}{\partial t}+z\cdot\partial b \qquad 4$$

With (3) we can direct operation of ∂b in new coordinates.

$$\frac{\nabla b}{\nabla t} = \frac{\partial b}{\partial t} + q_r \frac{\partial b}{\partial r} + q_\vartheta \frac{\partial b}{r\partial \theta} + q_z \frac{\partial b}{\partial z},$$

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$$\frac{\nabla b}{\nabla t} = \frac{\partial b}{\partial t} + q_r \frac{\partial b}{\partial r} + q_\vartheta \frac{\partial b}{r\partial \theta} q_\theta \frac{\partial b}{r \sin \theta \partial \theta}$$

Which the law of conservation of mass has already been presented in a form applicable to a control volume may be rewritten as:

$$(1) = \int_v \frac{\partial R}{\partial t} \partial v + \int_s \partial q \cdot nds \qquad 6$$

Application of the divergence

(2) theorem to the surface integral

$$\int_s pq \cdot nds = \int_v \Delta \cdot (pq) dv \cdot \qquad 7$$

Apply 6 into

$$\int_v \left[\frac{\partial p}{\partial t} = \Delta \cdot (pq) \right] dv = 0 \qquad 8$$

Hence, $\frac{\partial p}{\partial t} + \Delta(pq) = 0$ 9

The equation (9.0) is known as the equation f continuity. It is the differential form of the law of conservation of mass written in form of the flow field.

Equation (9.0) is now rewritten in detail in the three most continuity used coordinate systems.

In Cartesian coordinates

$$\frac{\partial p}{\partial t} + \frac{\partial (pu)}{\partial x} + \frac{\partial (pw)}{\partial y} + \frac{\partial (pw)}{\partial z} = 0 \qquad 10$$

In cylindrical coordinates

$$\frac{\partial p}{\partial t} + \frac{d}{r} \frac{\partial (rpqr)}{\partial r} + \frac{1}{r} \frac{\partial (pq\theta)}{\partial \theta} + \frac{\partial (pqz)}{\partial z} = 0 \qquad 11$$

In spherical coordinates

$$\frac{\partial p}{\partial t}+\frac{1}{r^2}\frac{\partial(r^2 pq)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial(pq\theta\sin\theta)}{\partial\theta}+\frac{1}{\sin\theta}\frac{\partial(pq\theta)}{\partial\theta}=0 \qquad 12$$

In some particular cases equation of continuity assumes simpler form given in Cartesian coordinates.

$$\left.\begin{array}{l}\Delta\cdot(pq)=0\\ or\\ \Delta\cdot q=0\end{array}\right\}. \qquad 13$$

Now, for momentum, Newton's second law of motion states that the rate of change of momentum of a thermodynamics system equals the sum total of the forces acting on the system.

$$\frac{D}{Dr}\int_v pqdv=\int_v gpdv+\int_s Tds \qquad 14$$

When g is a general body force per unit mass,

and T is the system boundary for *x*-component Equation 14 becomes

$$\frac{D}{Dt}\int_v pxdv=\int_v gxpdv+\int_s Tnxds \qquad 15$$

The Reynolds transport theorem may now be applied to the left-hand side of this equation

$$\frac{D}{Dt}\int_v pudv=\int_v \rho\left[\frac{du}{\partial t}+\frac{u\partial u}{\partial u}+\frac{v\partial u}{\partial y}+\frac{w\partial u}{\partial z}\right]dv \qquad 16$$

The stress term *Tu* inside the surface integral is now written in terms of its components to yield

$$\int_s T_{nn}ds=\int_s\left[T_{xxi}+T_{ynj}+T_{zx}k\right]\cdot nds=\int_v\left[\frac{\partial\tau_{xx}}{\partial u}+\frac{\partial\tau_{yx}}{\partial y}+\frac{\partial\pi t}{\partial_x}\right]dv \qquad 17$$

where the divergence theorem has been used again

By subtraction of equation 16 and 17 into equation 15 yields

$$\int_v\left\{\rho\left[\frac{\partial u}{\partial t}+\frac{u\partial u}{\partial u}+\frac{v\partial u}{\partial t}+\frac{w\partial u}{\partial t}-g_x\right]-\left[\frac{\partial T_{xx}}{\partial u}+\frac{\partial T_{yn}}{\partial y}+\frac{\partial T_{zx}}{\partial z}\right]\right\}\partial v=0 \qquad 18$$

Becomes

$$\rho\left[\frac{\partial u}{\partial t}+\frac{u\partial u}{\partial x}+\frac{v\partial u}{\partial y}+\frac{\omega\partial u}{\partial t}\right]=Pg_x+\frac{\partial T_{xx}}{\partial x}+\frac{\partial T_{yx}}{\partial y}+\frac{\partial T_{zx}}{\partial z} \qquad 19$$

Similarly for the y- and Z- Components

$$\rho\left[\frac{\partial u}{\partial t}+\frac{u\partial v}{\partial x}+\frac{v\partial u}{\partial y}+\frac{\omega\partial u}{\partial t}\right]P_{g_x}+\frac{\partial T_{xx}}{\partial x}+\frac{\partial T_{yx}}{\partial y}+\frac{\partial T_{zx}}{\partial z} \qquad 20$$

$$\rho\left[\frac{\partial u}{\partial t}+\frac{u\partial v}{\partial x}+\frac{v\partial u}{\partial y}+\frac{\omega\partial u}{\partial t}\right]=P_{g_x}+\frac{\partial T_{xx}}{\partial x}+\frac{\partial T_{yx}}{\partial y}+\frac{\partial T_{zx}}{\partial z} \qquad 21$$

The . ∇ g term vanishes for incompressible flows. Hence the thermodynamic presume may be define for an incompressible fluid as the average normal stress:

$$\rho=\frac{T_{xx}+T_{yy}+T_{zz}}{3} \qquad 22$$

It is customary to separate out the pressure terms from the total stress

$$.\mathrm{Tij}+-\mathrm{p}\partial ij+\tau ij \qquad 23$$

And $\tau_{ij}=\partial\mu\Sigma ij$ equation 23 is written in tensor form as

T=-p + τ 24

$$P=\begin{vmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{vmatrix}$$

Equation 23 is used to modify the momentum equation by subtraction 8 from 20.

$$\rho=\left[\frac{\partial u}{\partial t}+U\frac{\partial u}{\partial x}+V\frac{\partial u}{\partial y}+W\frac{\partial u}{\partial t}\right]=-\frac{\partial \rho}{\partial x}+\rho_{gy}+\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{yx}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z} \qquad 25$$

$$\rho\left[\frac{\partial u}{\partial t}+U\frac{\partial v}{\partial x}+V\frac{\partial v}{\partial y}+W\frac{\partial v}{\partial t}\int=-\frac{\partial p}{\partial x}+\rho_{gy}+\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \tau_{yx}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z}\right. \qquad 26$$

$$\rho\left(\frac{\partial w}{\partial t}+U\frac{\partial w}{\partial x}+V\frac{\partial w}{\partial y}+W\frac{\partial w}{\partial t}\right)=-\frac{\partial p}{\partial y}+\rho_{gy}+\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}+\frac{\partial \tau_{zz}}{\partial z} \qquad 27$$

This may be put in symbolic compact form.

$$\rho\frac{\Delta q}{\Delta t}=-\nabla_{p}\rho g+\nabla.\tau \qquad 28$$

The expression for the stress and the rate of strain component in several coordinate system are now written down.

In Cartesian coordinates of q=iu + jv + kw

$$\varepsilon_{xy}=\frac{1}{2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right),\varepsilon_{xy}=\frac{1}{2}\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right),\varepsilon_{xy}=\frac{1}{2}\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right),$$

$$\lambda_{xy}=\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right),\lambda_{xy}=\mu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right),\lambda_{xy}=\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right),$$

$$\varepsilon_{xx}=\frac{\partial u}{\partial x},\quad \varepsilon_{yy}=\frac{\partial v}{\partial y},\quad \varepsilon_{zz}=\frac{\partial w}{\partial r},$$

$$\lambda_{xx}=\partial\mu\frac{\partial u}{\partial x},\quad \lambda_{yy}=\partial\mu\frac{\partial r}{\partial y},\quad \lambda_{zz}=\partial\mu\frac{\partial w}{\partial z}, \qquad 29$$

In cylindrical coordinates $q=e_r q_r + e_9 q_9 + e_z q_z$

$$\varepsilon_{R9}=\frac{1}{2}\left[\frac{1}{r}\frac{\partial qr}{\partial \vartheta}+r\frac{\partial}{\partial r}\left(\frac{q^{\vartheta}}{r}\right)\right],\qquad \tau_{R9}=\mu\left[\frac{1}{r}\frac{\partial qr}{\partial 9}+r\frac{\partial}{\partial r}\left(\frac{q^{\vartheta}}{r}\right)\right]$$

$$\varepsilon_{r9}=\frac{1}{2}\left[\frac{\partial qr}{\partial \vartheta}+\frac{\partial}{\partial r}\right],\quad \tau_{rz}=\mu\left[\frac{\partial qr}{\partial z}+\frac{\partial z}{\partial r}\right]$$

$$\varepsilon_{9z}=\frac{1}{2}\left[\frac{\partial q\vartheta}{\partial \vartheta}+\frac{1}{r}\frac{\partial qz}{\partial \vartheta}\right],\qquad \tau_{9z}=\mu\left[\frac{\partial q\vartheta}{\partial z}+\frac{1}{r}\frac{\partial qz}{\partial \vartheta}\right]$$

$$\varepsilon_{rr}=\frac{\partial qr}{\partial r}\qquad \varepsilon_{9ee}=\left(\frac{1}{r}\frac{\partial q\vartheta}{\partial \vartheta}+\frac{qr}{r}\right),\qquad \varepsilon_{zz}=\frac{\partial qz}{\partial z}$$

$$\tau_{rr}=2\mu\frac{\partial qr}{\partial r},\qquad \tau_{99}=2\mu\left(\frac{1}{r}\frac{\partial q\vartheta}{\partial \vartheta}+\frac{qr}{r}\right),\ \tau_{zz}=2\mu\frac{\partial qz}{\partial z} \qquad 30$$

In spherical coordinates $q=e_r q_r + e_9 q_9 + e_{\phi} q_{\phi}$

$$ER\theta = \frac{1}{2}\left[\frac{1}{R}\,\partial qR\Big/\partial\theta + R\left(\frac{\partial}{\partial R}\right)\right], \tau R\theta = \mu\left[\frac{1}{R}\,\partial qR\Big/\partial\theta + R\left(\frac{\partial}{\partial R}\right)\right]$$

$$ER\varphi = \frac{1}{2}\left[\frac{1}{RSin\theta}\,\partial qR\Big/\partial\theta + R\left(\frac{\partial}{\partial R}\right)\right], \tau R\theta = \mu\left[\frac{1}{RSin\theta} + R\,\partial q\Big/\partial R\left(\frac{q^{\theta}}{\partial R}\right)\right]$$

$$E\theta\varphi = \frac{1}{2}\left[\frac{1}{RSin\theta}\,\partial qR\Big/\partial\theta + \frac{Sin\theta}{R}\,\partial\Big/\partial\theta\left(\frac{q\varphi}{Sin\theta}\right)\right], \tau R\theta = \mu\left[\frac{1}{RSin\theta}\,\partial q\theta\Big/\partial\varphi + \frac{Sin\theta}{R}\,\partial\Big/\partial R\left(\frac{q^{\theta}}{Sin\theta}\right)\right]$$

$$\text{ERR}=\frac{\partial qR}{\partial R}, \quad \tau_{RR} = 2\mu\frac{\partial qR}{\partial R},$$

$$E\theta\theta = \left(\frac{1}{R}\frac{\partial qR}{\partial R} + \frac{qr}{R}\right), \qquad \tau\theta\varphi = 2\mu\left(\frac{1}{R}\frac{\partial qR}{\partial R} + \frac{qr}{R}\right)$$

$$E\varphi\varphi = \left(\frac{1}{RSin\varphi}\frac{\partial q\varphi}{\partial\varphi} + \frac{qR}{R}\frac{q\theta Cot\theta}{R}\right), \qquad \tau\varphi\varphi = 2\mu\left(\frac{1}{RSin\varphi}\frac{\partial q\varphi}{\partial\varphi} + \frac{qR}{R}\frac{q\theta Cot\theta}{R}\right) \quad 31$$

Equation 29-31 may be used to eliminate the stress components from the differential momentum equation 25-27

Becomes

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \rho_{gx}\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \rho_{gw}\frac{\partial p}{\partial w} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = \rho_{gw}\frac{\partial p}{\partial w} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) \qquad 32$$

Equ. 32 constitute a system of three nonlinear second order partial differential equation. The proper boundary conditions for the velocity on a rigid boundary are: $q_n = q_t = 0$

Where q_n is the minimal component of the velocity relative to the solid boundaries and q_r is its tangential component. These conditions are also termed the non-peneuration (q_n = 0) and no-slip (q_t = 0) viscous boundary conditions.

Equation 3.2 becomes

$$\rho\frac{Dq}{Dt} = -\nabla\rho + \rho g - \mu\nabla x(\nabla xq) = -\nabla\rho + \rho g - \mu\nabla^2 q \qquad 33$$

If ∇^2 is the Laplacian operator applied to the velocity vector in Cartesian coordinates. By expanding $\nabla \times (\nabla \times q)$ in cylindrical polar coordinates and using (13) we obtain

$$\rho\left(\frac{\partial qr}{\partial t} + q_r \frac{\partial qr}{\partial r} + q\theta \frac{\partial qr}{r\partial \theta} + q_z \frac{\partial qr}{\partial z} - \frac{q^2\theta}{r}\right)$$

$$= \rho_{gr} - \frac{\partial p}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\right)(rq_r)\right] + \frac{1}{r^2}\frac{\partial^2 q_r}{\partial z^2} - \frac{p}{r^2}\frac{\partial q\theta}{\partial \theta} \qquad 34$$

$$\rho\left(\frac{\partial q\theta}{\partial t} + q_r \frac{\partial q\theta}{\partial r} + q\theta \frac{\partial q\theta}{r\partial \theta} + q_z \frac{\partial q\theta}{\partial z} - \frac{qrq\theta}{r}\right)$$

$$= \rho_{g\theta} - \frac{\partial p}{r\partial \theta} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rq\theta)\right) + \frac{1}{r^2}\frac{\partial^2 \theta q_r}{\partial z^2} + \frac{\partial^2 \theta q_r}{\partial z^2} + \frac{2}{r^2}\frac{\partial q\theta}{\partial \theta}\right] \qquad 35$$

$$\rho\left(\frac{\partial qz}{\partial t} + qr\frac{\partial qz}{\partial r} + q\theta \frac{\partial qz}{r\partial \theta} + q_z \frac{\partial qz}{\partial z}\right)$$

$$= \rho_{yz} - \frac{\partial p}{\partial y} + \mu\left[\frac{I}{r}\frac{\partial}{\partial r}\left(r\frac{\partial qz}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 qz}{\partial \theta^2} + \frac{\partial^2 qz}{\partial z^2}\right] \qquad 36$$

By repeating the for spherical coordinate, we obtaqin,

$$= \rho_{y\varphi} - \frac{1}{RSin^2\theta}\frac{\partial p}{\varphi} + \mu\left[\frac{I}{R}\frac{\partial}{\partial R}\left(R\frac{\partial q\theta}{\partial R}\right) + \frac{1}{R^2 Sin\theta}\frac{\partial^2}{\partial \theta^2} + \left(Sin\theta\frac{\partial q\varphi}{\partial \theta}\right)\right]$$

$$+ \mu\left[\frac{I}{RSin^2\theta}\frac{\partial^2 q\varphi}{R^2 Sin^2\theta} + \frac{2}{R^2 Sin\theta}\frac{\partial qr}{\partial \varphi} + \frac{2Cos\theta}{R^2 Sin^2\theta}\frac{\partial q\varphi}{\partial \varphi}\right] \qquad 37$$

By substituting $\mu = 0$ in the navier-stokes equating which is called momentum equation (3.2) – (3.7) we obtain an equation

$$\rho\frac{Dq}{Dt} = \rho_g - \nabla\rho \qquad 38$$

This is called the Euler equation

3.0 Discussion

Solutions of the momentum equation result in velocity vectors q and pressure ρ which satisfy both the momentum equation and the continuity equation. Given such a combination, [$q \,.\, \rho$], we can check whether it constitutes a solution by substitution into the equations. How to find such a solution is another matter and any general step leading toward this goal is useful. For two dimensional flows it is possible to eliminate the continuity equation from the system of equations by using only functions which satisfy the continuity equation. This elimination is a formal step toward a solution and functions which affect this elimination and the stream functions.

And if the flow is defined as two dimensional when its description in Cartesian coordinates shows no z-component of the velocity and no dependence on the z-coordinate. Such a flow can be described in the $z=0$ plane, show a flow pattern identical to that in the $z=0$ plane. The $z=0$ plane is therefore called representative plane.

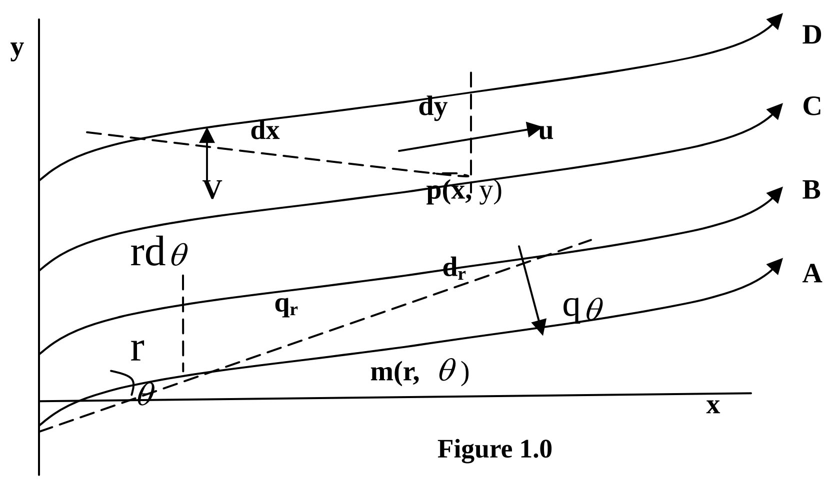

Figure 1.0

The figure 1.0 shows a representative plane for two-dimensional flow, with four streamlines denoted by the letters A, B, C, D. the whole pattern may be shifted

In the z-direction parallel to itself. Thus the streamlines also represent stream sheets, i.e barriers which are not crossed by the flow. The Mass flux entering at the left, between, say, streamlines A and B must therefore come out at the right side without change. Because the distance between the two streamlines accommodating this mass flux seems in the drawing to increase, the mass flux seems per unit Cross section $\rho..q$, must decrease from left to right. There is therefore some relation between the convergence and divergence of stream lines and the vector $\rho.q$. Furthermore, because stream sheets are not crossed by the flow, each sheet represents a certain mass flux per unit depth of stream sheet taking place below it i. flowing between it and some particular stream sheet representing zero flux.

This mass flux is called the stream function and it is denoted by φ

$$\partial\varphi = (\partial y)(up)$$

$$= (-\partial x)(vp)$$

From which follows

$$u\lambda = \frac{\partial x}{\partial y}, \quad v\rho = \frac{-\partial\varphi}{\partial x}$$

Using planne polar coordinate in the representative plane and letting.

$$\varphi B = \varphi A + d\varphi$$

$$d\varphi = (rdq(q_r\rho)$$

$$d\varphi = (dr)(-qq\rho)$$

From which follows

$$q_r\rho = \frac{1}{r}\frac{\partial\varphi}{\partial\theta}, \qquad q\theta\rho = \frac{-\partial\varphi}{\partial r}$$

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